PRINCIPAL STRESSES AND STRAINS

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The Concept and definition of stress, strain, types of stresses (ie Tensile, Compressive and Simple shear) and types of strain (ie tensile, Compressive, shear & Volumetric strain) are discussed.

These stresses were acting in a plane which was at right angles to the line of action of force.

In many Engineering problems both direct (Tensile or Compressive stress) and shear stresses are acting at the same time. In such situations the resultant stress across any section will be neither normal nor tangential to the plane. In this chapter the stresses acting on an inclined plane (or oblique section) will be analysed.

A general stress system in the case of three-dimensional stresses can be represented by three normal stresses and three shear stresses

The system in which direct stresses and shearing stresses act simultaneously are called Combined (or) Compound stresses.

PRINCIPAL planes and principal stresses

The planes, which have no shear stress, are known as principal planes. Hence principal planes are planes of zero shear stress. Those planes carry only normal stresses.

The normal stresses, acting on a principal plane, are known as principal stresses.

Methods for determining stresses on oblique section

1. Analytical Method 2. Graphical Method.

1. Analytical Method for determining Stresses on oblique section:

(*) & We have considered only the normal stresses acting on c/s, such as on c/s mn of a bar AB (Fig a) Analysis is made on the stresses induced on the inclined sections such as section pq

In actual practice, all the three types of stresses [Tension, Compr & Shear] may act simultaneously on appropriate planes passing through a point in the strained material. Such a General system is known as

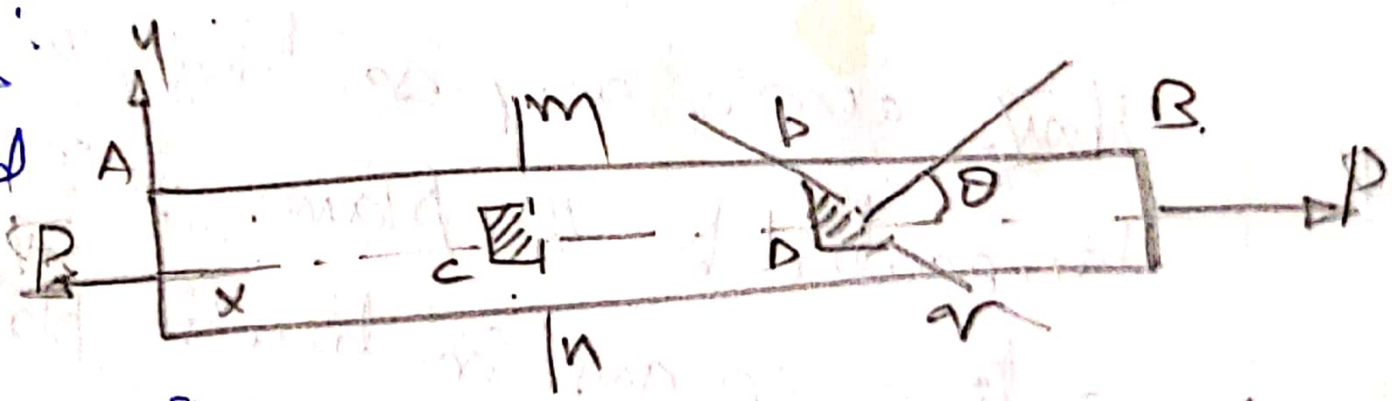


Fig 1
(a) Bar subject to uniaxial stress

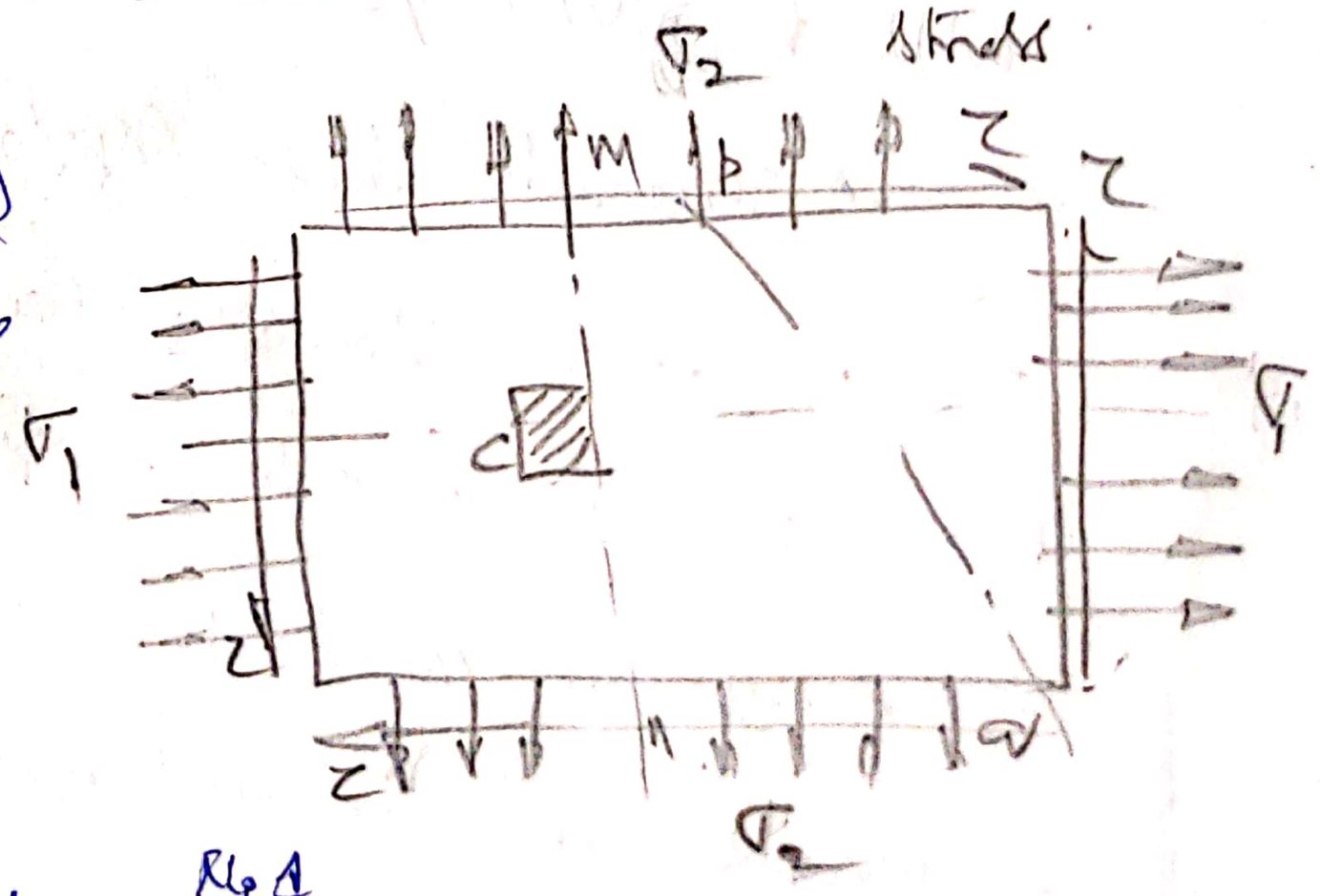


Fig 1
(b) Body subject to General Stress system

STRESSES INDUCED DUE TO UNIAXIAL STRESS

a) Stresses on normal C/S mn:

Consider C/S mn of a bar AB [Fig 1(a)] Sub. to uniaxial tensile force P.

Let us assume that the stress distribution is uniformly over the entire cross sectional area. This is possible when the following conditions exist.

1) The bar is prismatic 2. The material is homogeneous and isotropic 3) The force acts @ the centroid of C/S area 4) The C/S is away from the ends of the bar where higher localised stresses may exist.

In this case σ = stress = $\frac{P}{A}$.

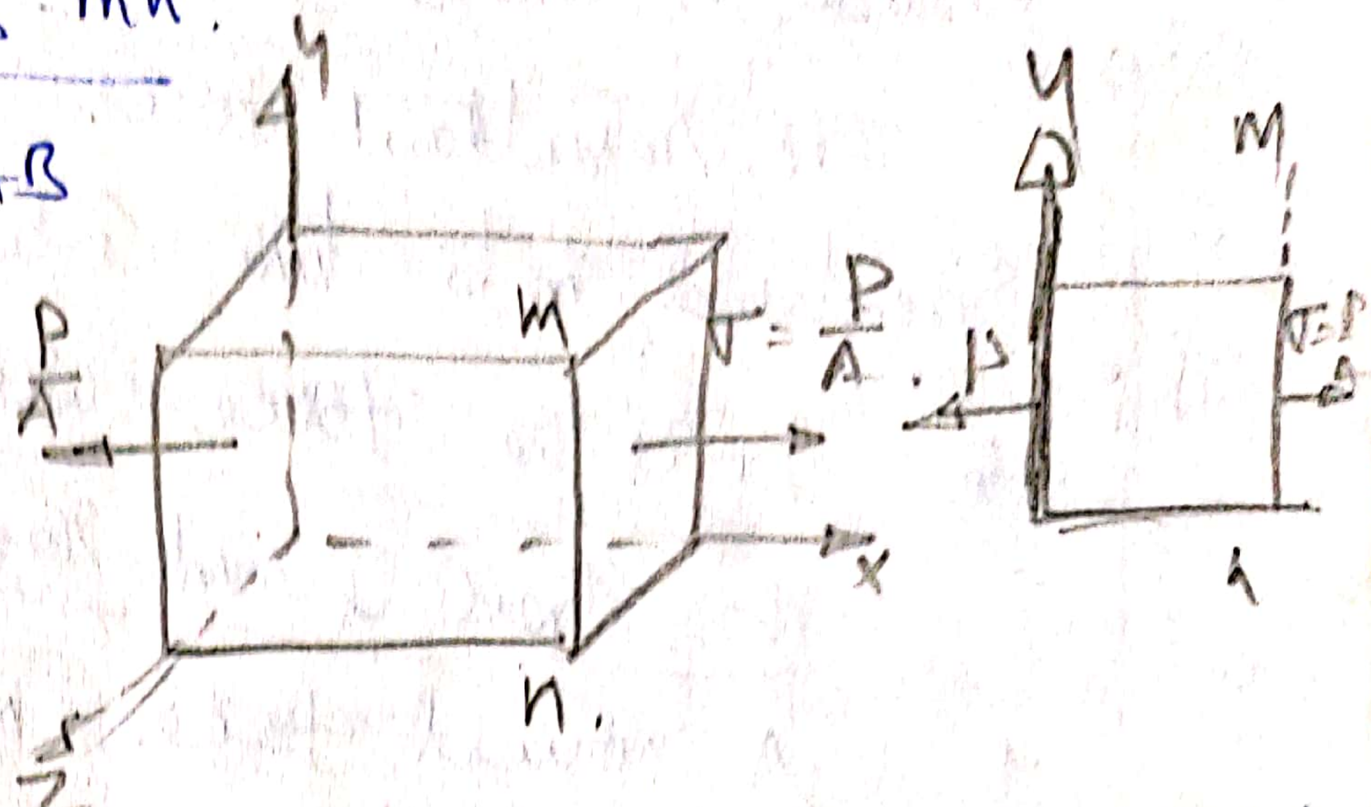


Fig2 Stress element At Point

b) Stresses on inclined section pq

Consider an inclined section pq of a bar AB [Fig 1(a) & Fig 3(a)] Sub. to uniaxial tensile force P. This inclined section pq is cut through the bar at an angle θ b/t the x axis and the normal to the plane pq.

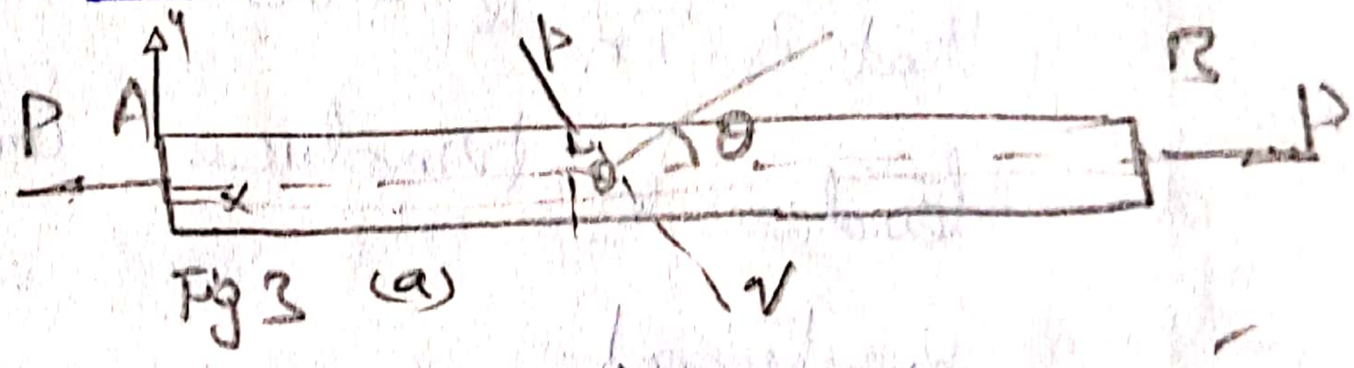
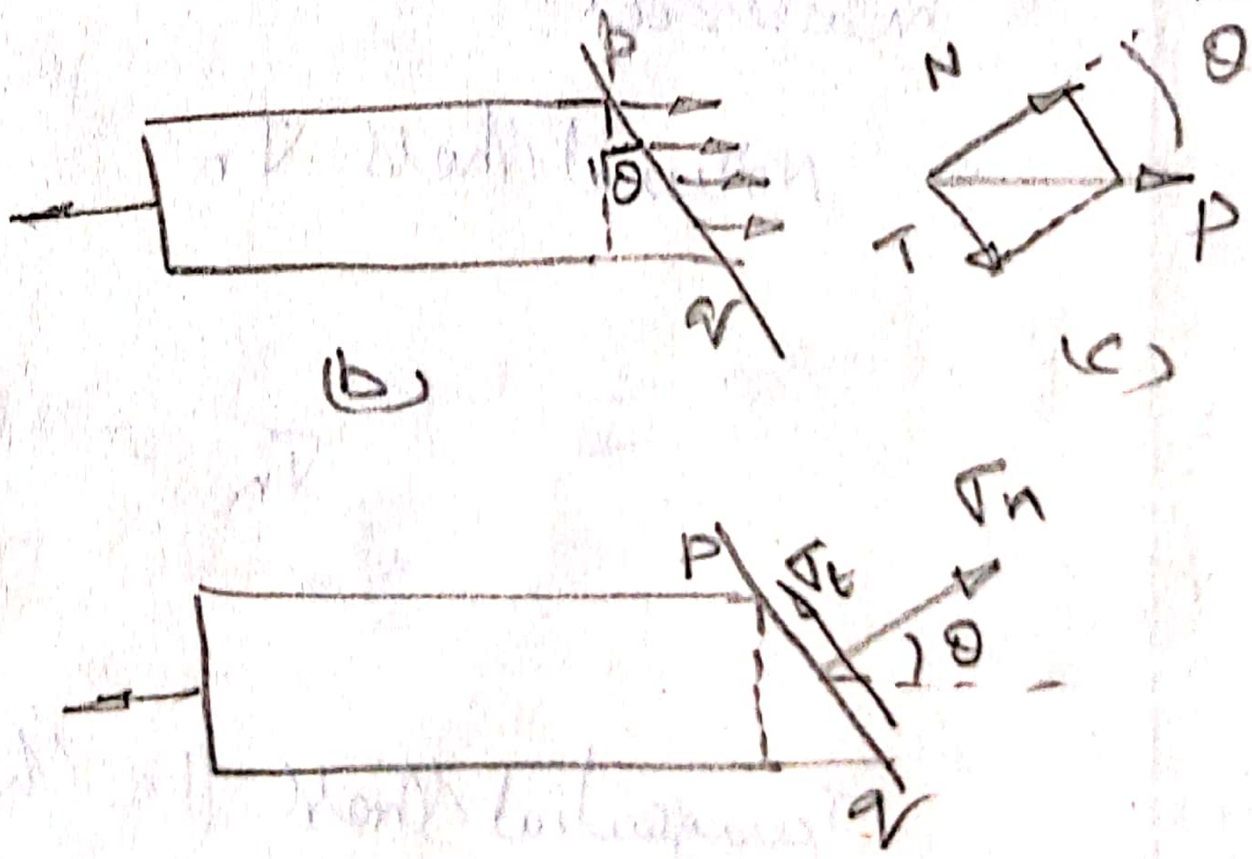


Fig 3 (a)



(b) (c)

Since all the parts of the bar have the same axial stress the stresses acting on the C/S pq are uniformly distributed.

The resultant of these stresses will be force equal in magnitude to axial force P.

This resultant force acting on section pq may be resolved in to two components. Fig 3(c)

Note: It is the force which is resolved in two components. The stress is not resolved.

1. A normal force N acting normal to the plane pq
2. A tangential force T " tangential to "

$\therefore$ $N = P\cos\theta$ and $T = P\sin\theta$ [From Fig 3(c)]

$\Rightarrow$ These force components N & T give rise to normal stress σ_n & tangential stress σ_t resp. which are uniformly distributed over the inclined section pq

$\Rightarrow$ These stresses are shown acting in +ve direction that is σ_n is +ve in tension & σ_t is +ve when it tends to produce counter clockwise rotation of the material.

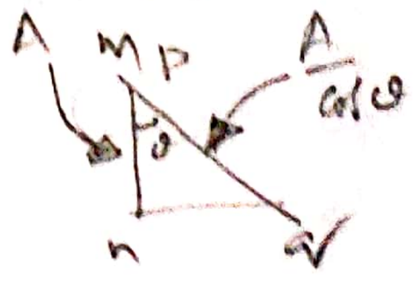
$\therefore$ Normal stress σ_n = $\frac{\text{Force normal to section pq}}{\text{Area of section pq}}$.

$$\sigma_n = \frac{P\cos\theta}{\frac{A}{\cos\theta}} = \frac{P}{A}\cos^2\theta = \sigma\cos^2\theta \quad \left[\because \frac{P}{A} = \sigma\right]$$

Tangential stress (i.e shear stress)

σ_t = $\frac{\text{Tangential force across section pq}}{\text{Area of section pq}}$

$$= \frac{P\sin\theta}{\frac{A}{\cos\theta}} = \frac{P}{A}\sin\theta\cos\theta = \frac{\sigma}{2}\sin 2\theta$$



[$\because$ By Area of section mn = A, Area of section pq = $\frac{A}{\cos\theta}$

The resultant stress $\sigma_r = \sqrt{\sigma_n^2 + \tau_t^2}$

$= \sqrt{(\sigma\cos^2\theta)^2 + (\frac{\sigma}{2}\sin 2\theta)^2} = \sigma\cos\theta$.

Note: 1.

From $\sigma_n = \sigma\cos^2\theta$, it is seen that the normal stress σ_n on the section pq will be more, when $\cos^2\theta$ or $\cos\theta$ is more, ie $\theta = 0^\circ$; But when $\theta = 0^\circ$, the section pq will coincide with section mn. But the section mn is normal to the line of action of the loading. This means the plane normal to the axis of loading will carry the more normal stress.

$\therefore$ More normal stress $= \sigma\cos^2\theta = \sigma\cos^2 0^\circ = \sigma$.

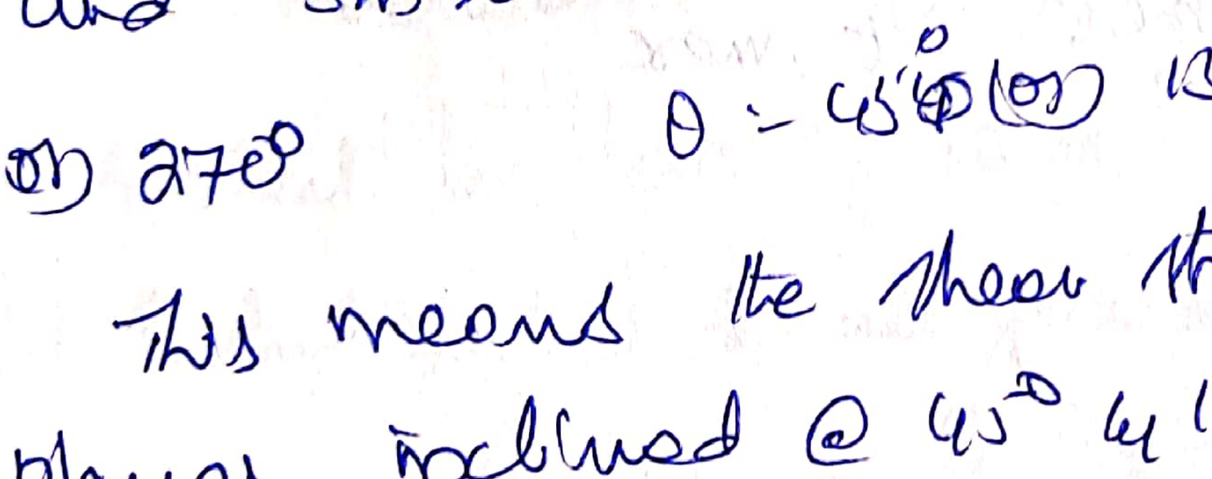
Note 2: Tangential stress (ie shear stress) across the section pq will be more when $\sin 2\theta$ is more. and $\sin 2\theta$ will be more when $\sin 2\theta = 1$ (or) $2\theta = 90^\circ$ or 270° $\theta = 45^\circ$ (or) 135°.

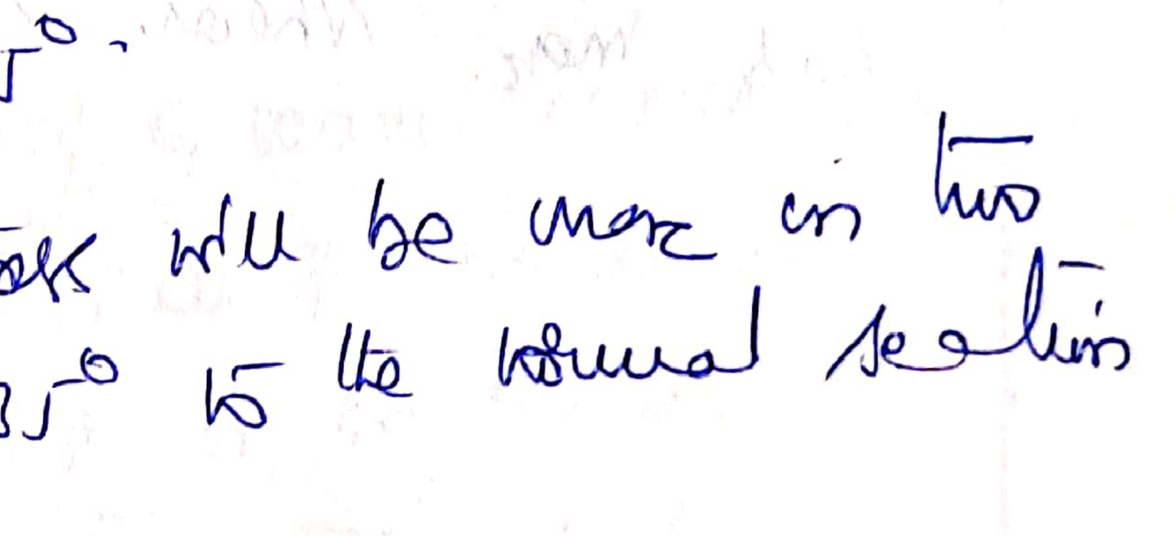
This means the shear stress will be more on two planes inclined @ 45° & 135° to the normal section mn as shown

$\therefore$ More value of shear stress $= \frac{\sigma}{2}\sin 2\theta = \frac{\sigma}{2}\sin 90 = \frac{\sigma}{2}$

$\therefore \tau_t = \frac{\sigma}{2}$

First plane of more shear stress $\theta = 45^\circ$

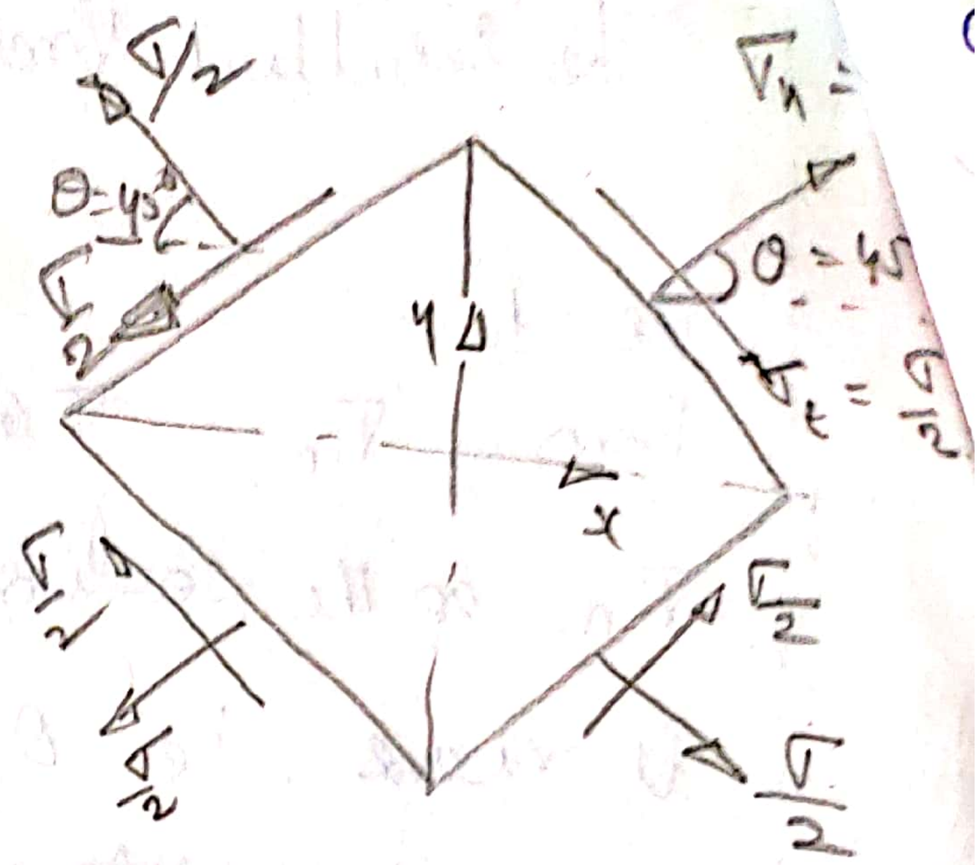




Second plane of more shear stress $\theta = 135^\circ$.

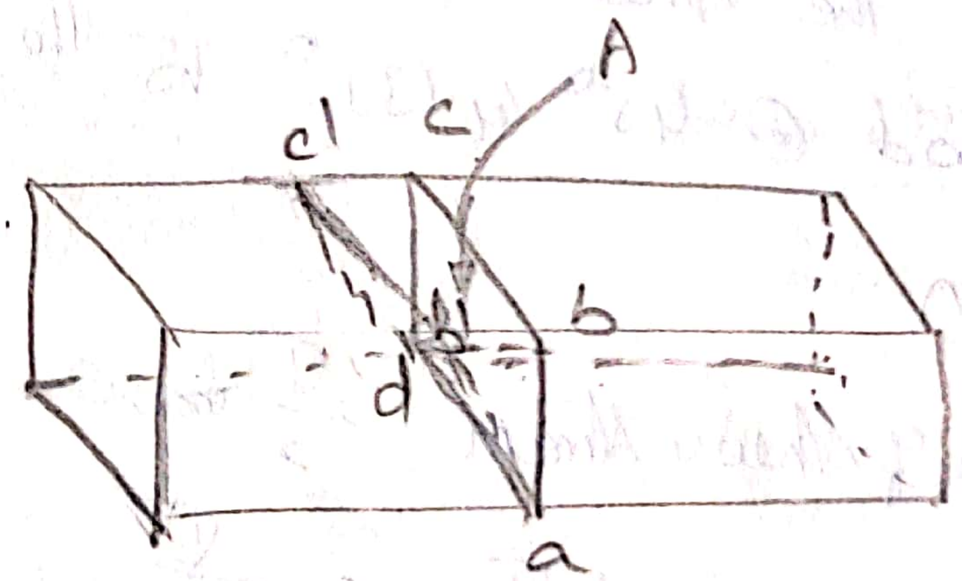
Note:- All the surfaces inclined at 45° to the axes of Bull are subjected to pure shear stress equal to $\sigma/2$

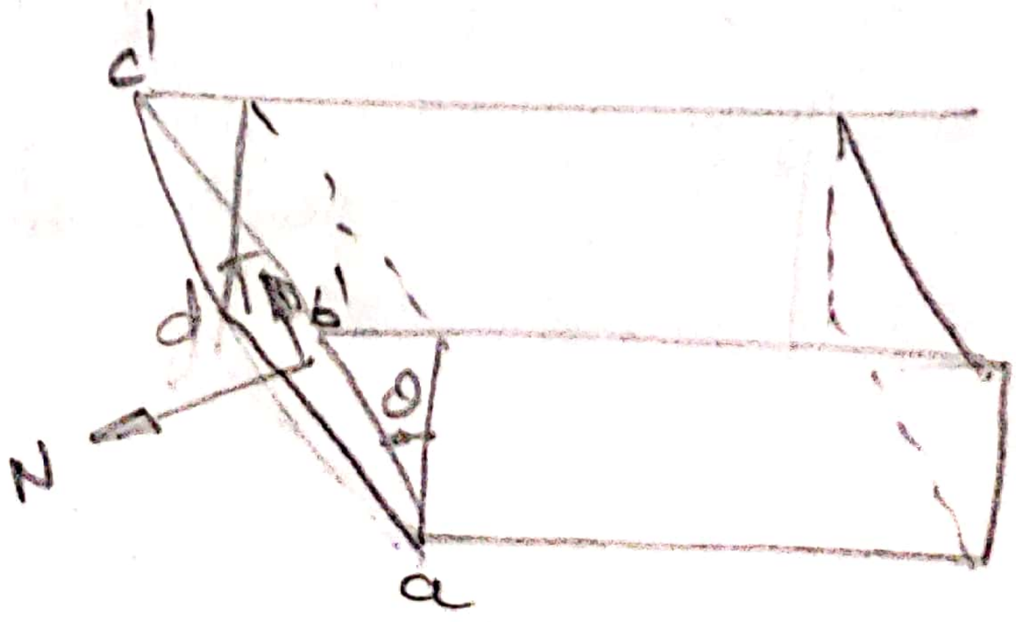
The complete state of stress on section cut at 45° to axis of Bull is represented by stress element shown



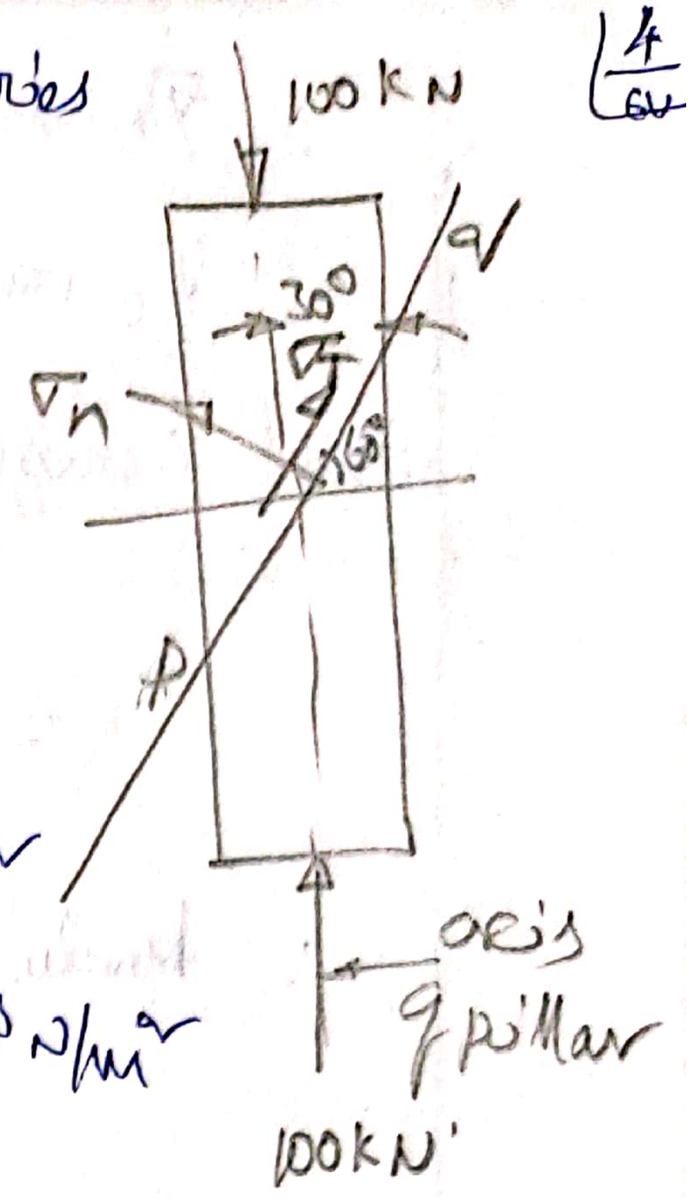
Fig(4). Stress element At θ = 45° For a Bar in Tension

Note: In the uni-axial stress system, where the bar is subjected only to simple tension (or) compression the two most important orientations of stress elements are θ = 0 and 45°, the former orientation has max normal stress $\sigma_{n,max}$ (Fig 2) & latter orientation has max shear stress $\sigma_{t,max}$ (Fig 4).





A short cast iron pillar of 625 mm² c/s carries an axial compressive load of 100 kN. Cal magnitudes of the normal & shear stresses on a plane inclined at 30° to the axis of the pillar. what is the value of max shear stress in the material



Normal area of the cross section = 625 mm²

$\therefore$ Direct stress $\sigma = \frac{100000}{625 \times 10^{-6}} = 160 \times 10^6 \text{ N/m}^2$

The plane PQ is inclined at 60° w.r.t X axis

$\therefore \theta = 60^\circ$ $\therefore$ Normal stress on inclined plane

$\sigma_n = \sigma \cos^2\theta = 160 \times 10^6 \cos^2 60^\circ = 40 \times 10^6 \text{ N/m}^2$

$\tau_t = \frac{\sigma}{2} \sin 2\theta = \frac{160 \times 10^6}{2} \frac{\sqrt{3}}{2} = 69.28 \times 10^6 \text{ N/m}^2$

Max shear stress $= \frac{\sigma}{2} = \frac{160 \times 10^6}{2} \text{ N/m}^2 = 80 \times 10^6 \text{ N/m}^2$

(10) A steel tie bar of 10 m long & 100 mm x 25 mm in CS. It is subjected to an axial pull of 300 kN and the increase in length is found to be 5 mm. Find

(i) the intensity of stress on a right cross section

(ii) The intensities of stress normal & tangential on a plane section inclined @ 60° to longitudinal axis

(iii) The modulus of elasticity of steel.

Sol: C/s area of the bar $= 100 \times 25 = 2500 \times 10^{-6} \text{ m}^2$

(i) Intensity of stress on right c/s. $= \frac{300 \times 1000}{2500 \times 10^{-6}} = 120 \text{ MN/m}^2$

(ii) Since the plane is inclined @ an angle of 60° to

longitudinal axis $\theta = (90 - 60) = 30^\circ$

$\sigma_1 = \sigma \cos^2\theta = 120\times10^6 \cos^2 30 = 90\times10^6$

σ_t (or) $\tau = \frac{\sigma}{2}\sin 2\theta = \frac{120\times10^6}{2}\cdot\frac{\sqrt{3}}{2} = 51.96\times10^6$

(iii) Longitudinal strain in the bar $= \frac{\text{Elongation}}{\text{original length}}$

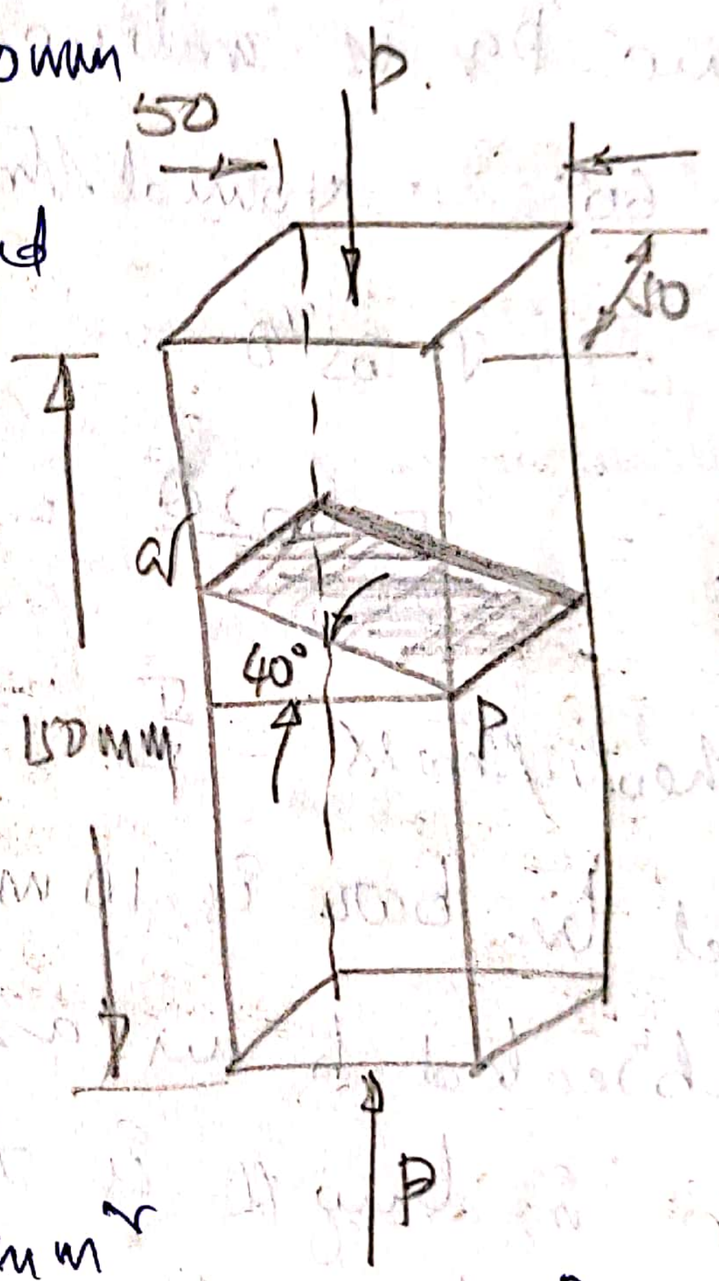
$= \frac{5}{10\times1000} = \frac{1}{2000}$.

$\therefore$ Modulus of elasticity $= \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\Delta} = \frac{120\times10^6}{1/2000} = 240\times10^6$ N/m²

$= 240$ GN/m²

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(12) A piece of wood 50×50×150 mm is sub. to a compressive load of P = 55 kN as shown. Find
(a) The normal & shearing stresses on the inclined plane, where $\theta = 40°$ &
(b) the max & normal stresses on a plane on which the shear stress is max.

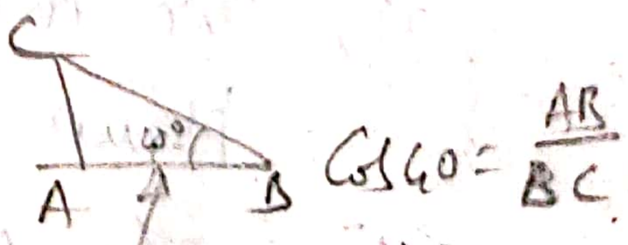


Sol: Normal area of c/s

$A = 50\times50 = 2500$ mm²

Intensity of stress on a normal c/s $\sigma_y = \frac{P}{A} = \frac{55000}{2500\times10^{-6}} = 22$ MN/m²

Inclined Area of c/s $= \frac{A}{\cos 40°}$



$\cos 40 = \frac{AB}{BC}$

$BC = \frac{AB}{\cos 40°}$

Normal Component of load on the inclined section par $= P\cos 40°$.

$\therefore$ Normal stress σ_n on the inclined plane. $= \frac{P\cos 40}{A/\cos 40°} = 12.9\times10^6$ N/m²

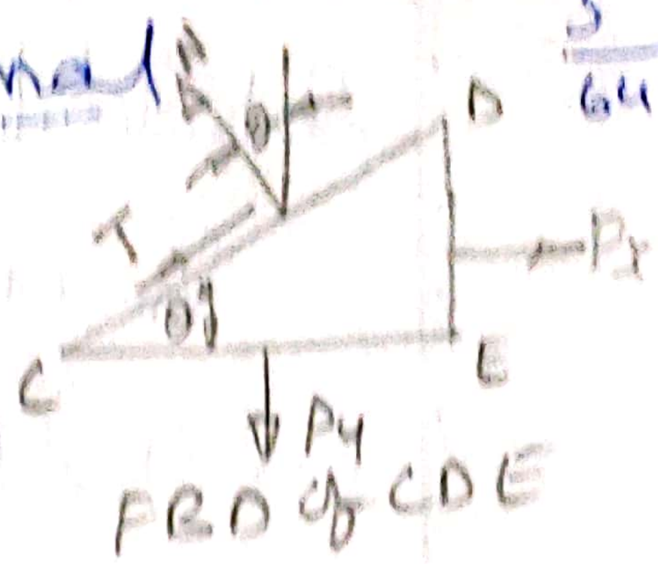
Tangential component of load $= P\sin 40°$.

τ (or) $\sigma_t = \frac{P\sin 40}{A/\cos 40} = \frac{P}{A}\sin 40\cos 40 = 10.83\times10^6$ N/m²

Max shear stress $= \frac{\sigma}{2} = \frac{22\times10^6}{2}$ normal $= \frac{\sigma_y}{2} = \frac{22\times10^6}{2} = 11$ MN/m²

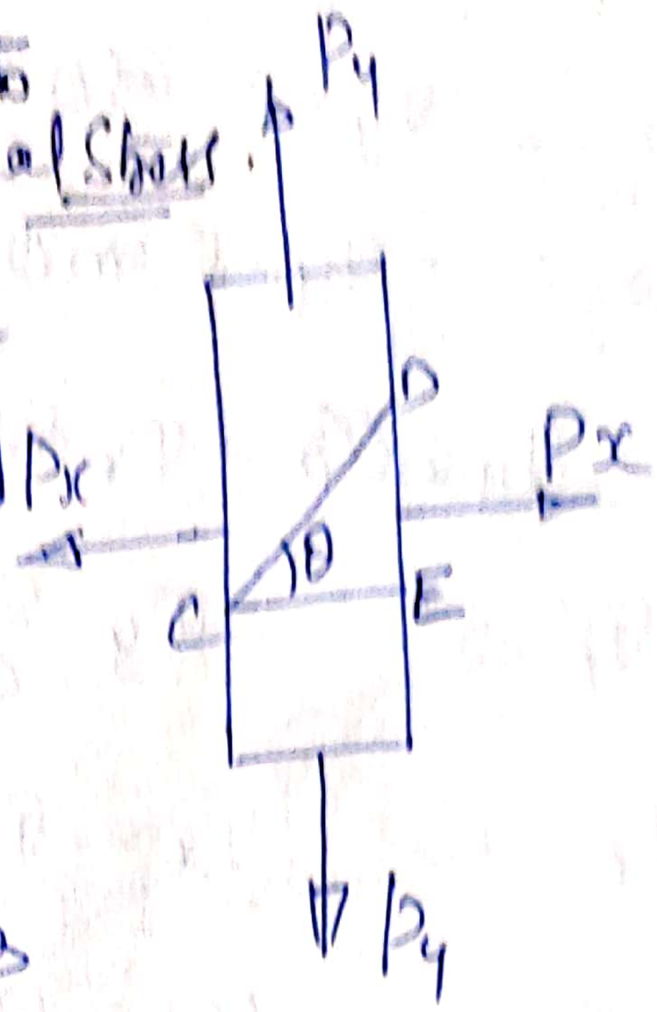
$= 11\times10^6$ N/m²

ELEMENT Subjected to two normal



Stresses: Stresses due to Bi-axial Stress.

Case (a) when both the stresses are like (Tensile)



In fig the bar is subjected to two direct tensile stresses σ_x & σ_y.

To find the normal & Tangential stress Components on an inclined plane CD. Let the plane CD be inclined at an angle θ to the horizontal.

Let N and T be the normal and tangential forces on the inclined plane CD.

Let σ_n and σ_t be the corresponding stresses.

$\therefore N = P_y \cos\theta + P_x \sin\theta$, $T = -P_y \sin\theta + P_x \cos\theta$. — (1)

Considering depth of bar as unity

$N = \sigma_n (CD \times 1)$; $P_x = \sigma_x (DE \times 1)$

$T = \sigma_t (CD \times 1)$; $P_y = \sigma_y (CE \times 1)$

[$\because$ Stress $\sigma = \frac{\text{Force}}{\text{Area}} \frac{P}{A}$]

Substituting the value in 1)

$$\sigma_n\, CD = \sigma_y\, CE \cos\theta + \sigma_x\, DE \sin\theta$$

$$\sigma_t\, CD = -\sigma_y\, CE \sin\theta + \sigma_x\, DE \cos\theta \quad \text{— (2)}$$

$$\therefore \sigma_n = \sigma_y \cos^2\theta + \sigma_x \sin^2\theta$$

Similarly $\sigma_t = -\sigma_y \frac{CE}{CD} \sin\theta + \frac{\sigma_x\, DE}{CD} \cos\theta.$ $\left[\because \frac{CE}{CD} = \cos\theta \;\; \frac{DE}{CD} = \sin\theta\right]$

$$\sigma_t = (\sigma_x - \sigma_y) \sin\theta\cos\theta$$

$$\therefore \sigma_t = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta \qquad \left[\because \theta = 45^\circ\right]$$

$\left[\sigma_{t|max} = \left(\frac{\sigma_x - \sigma_y}{2}\right)\right.$ The value of the normal tensile stress σ_n corresponding to the max shear stress value $\left.\text{when } \theta = 45^\circ\right]$

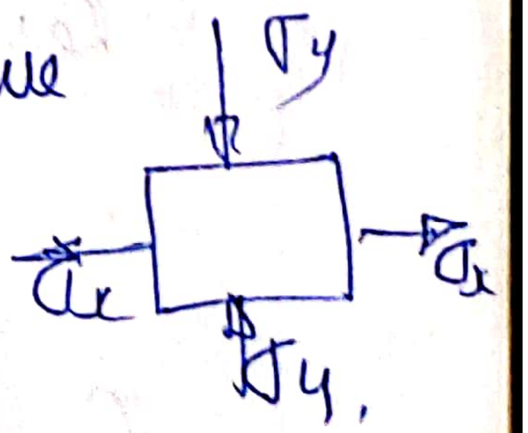
$$\sigma_n = \sigma_y \left[\frac{1}{\sqrt{2}}\right]^2 + \sigma_x \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{\sigma_x + \sigma_y}{2} \quad [\text{from (2)}]$$

If σ_R is resultant stress b/t σ_n & σ_t

$$\therefore \sigma_R = \sqrt{\sigma_x^2 \sin^2\theta + \sigma_y^2 \cos^2\theta} \qquad \left[\because \sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}\right]$$

Case(b) : When both stresses are unlike:

Let us take σ_x as tensile & σ_y as compressive



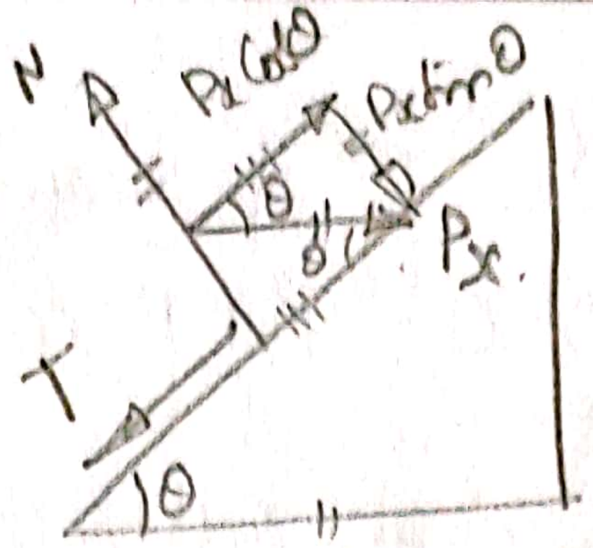
Normal stress $\sigma_n = \sigma_x \sin^2\theta + (-\sigma_y)\cos^2\theta$

$$\therefore \sigma_n = \sigma_x \sin^2\theta - \sigma_y \cos^2\theta$$

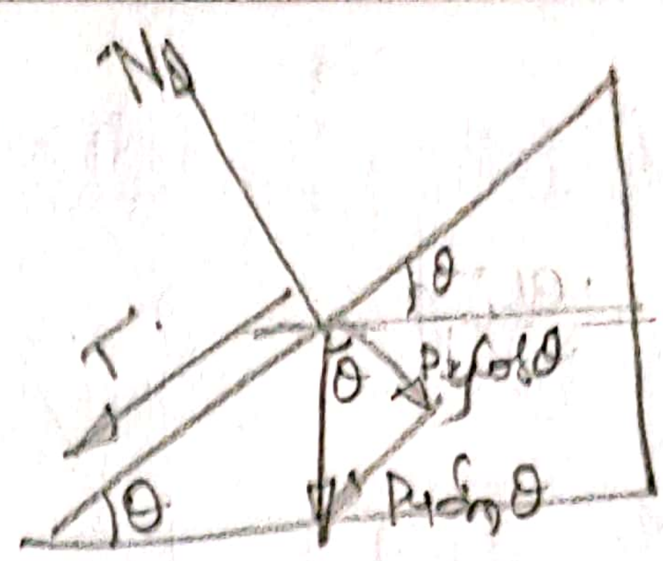
Tangential stress $\sigma_t = \frac{\sigma_x - (-\sigma_y)}{2} \sin 2\theta = \frac{\sigma_x + \sigma_y}{2} \sin 2\theta.$

$$\therefore \left(\sigma_t|_{max} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \sin 2\theta\right.$$

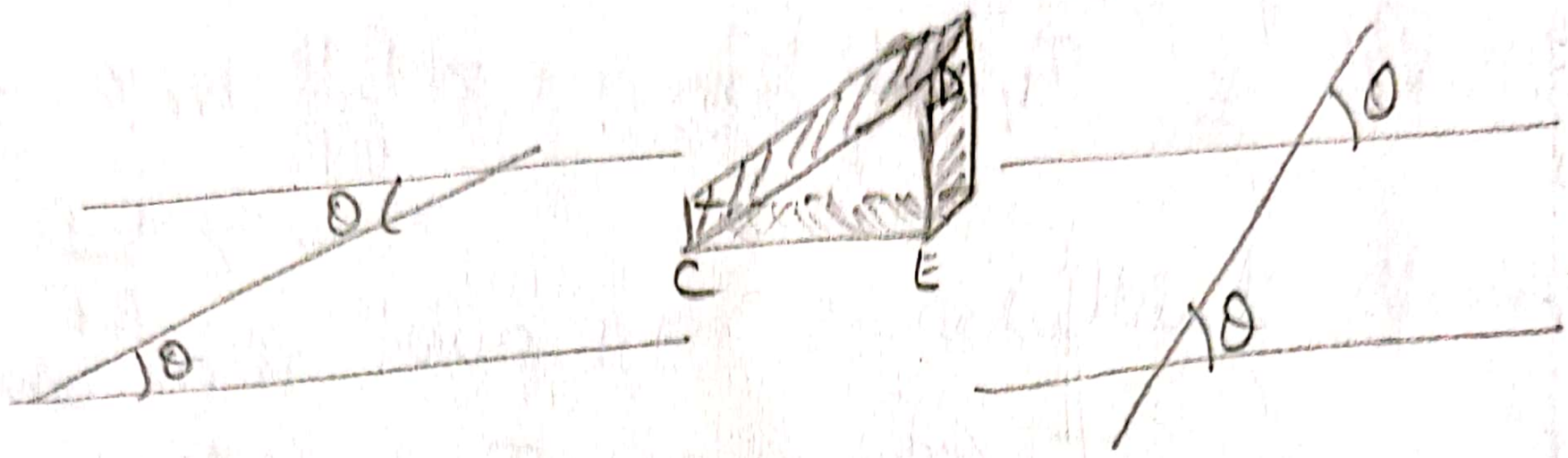
@ $\theta = 45^\circ$



Transfer of Px onto Inclined plane



Transfer of Py onto Inclined plane.



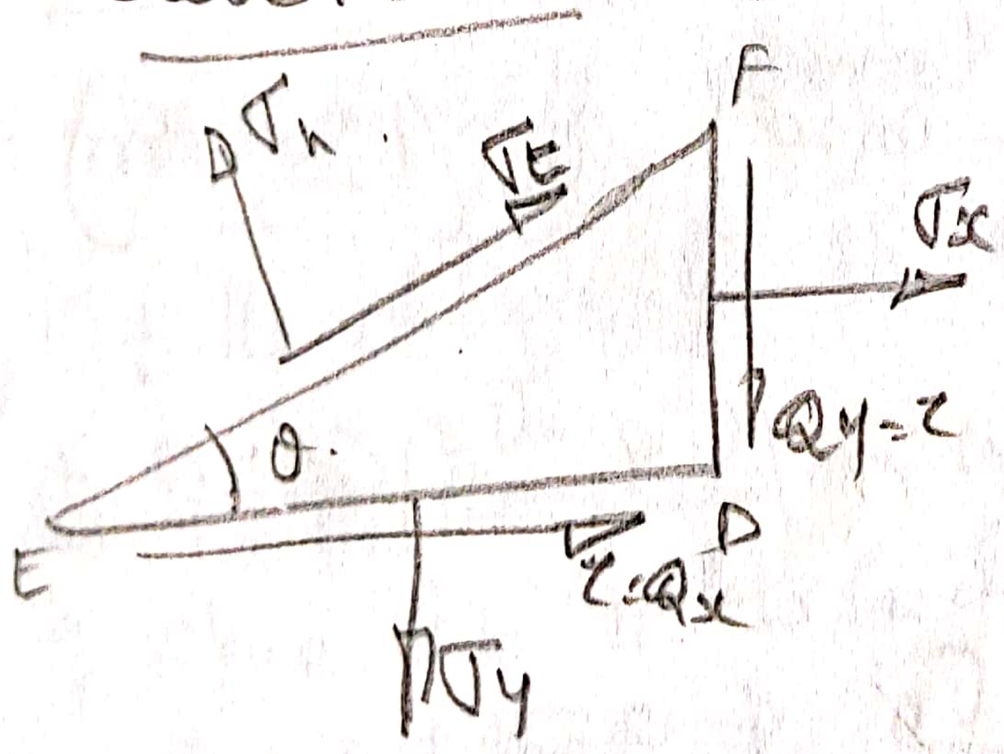
$\Sigma Y = 0 \qquad N - P_x \sin\theta - P_y \cos\theta = 0.$

$\therefore N = P_y \cos\theta + P_x \sin\theta \qquad -(1)$

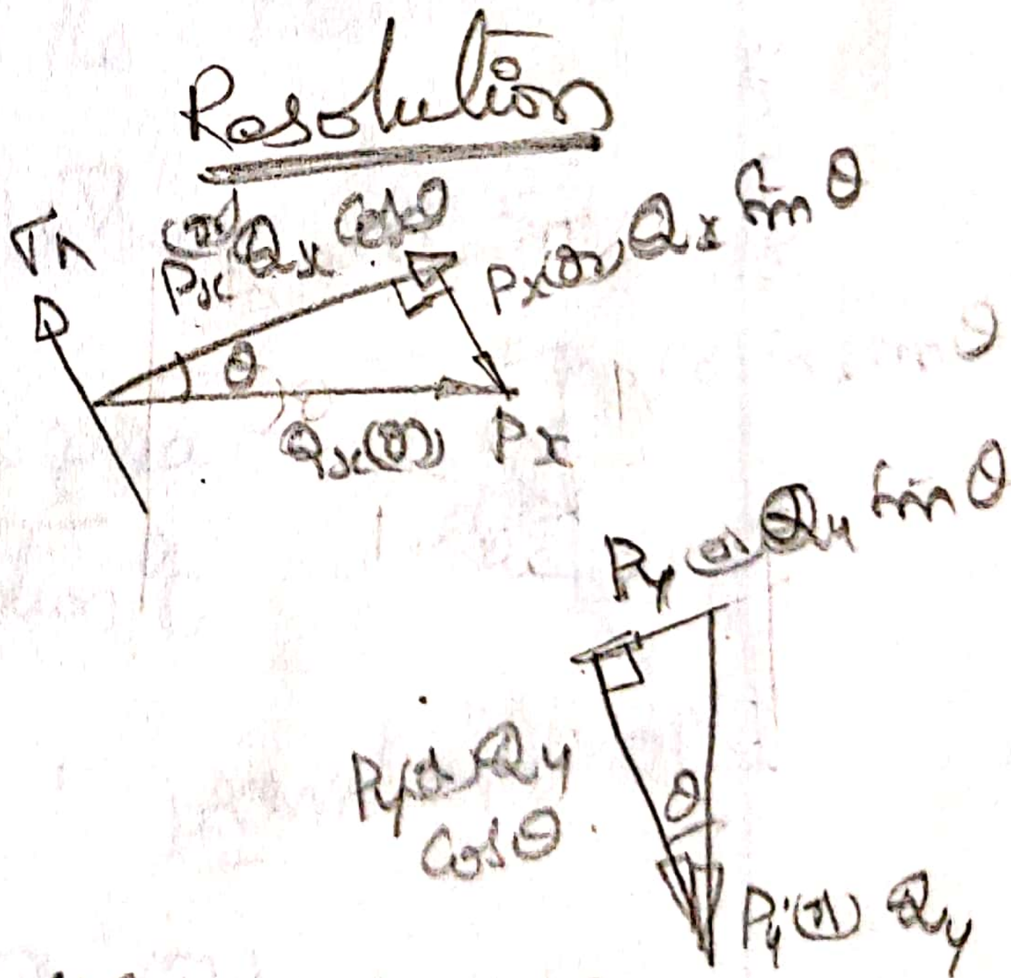
$\Sigma X = 0 \qquad -T - P_y \sin\theta + P_x \cos\theta = 0$

$T = -P_y \sin\theta + P_x \cos\theta \qquad -(2).$

General Two - DIMENSIONAL Stress System



Resolution



$\sigma_n \cdot EF = \sigma_x FD \sin\theta + \sigma_y ED \cos\theta + \tau ED \sin\theta + \tau FD \cos\theta.$

$\sigma_n = \sigma_x \sin\theta \frac{FD}{EF} + \sigma_y \cos\theta \frac{ED}{EF} + \tau \sin\theta \frac{ED}{EF} + \tau \cos\theta \frac{ED}{EF}$

$= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \sin\theta \cos\theta + \tau \cos\theta \sin\theta.$

$\qquad -(1)$

for $\frac{dU_h}{d\theta} = 0$.

$\therefore \quad \left(\frac{\nabla_y - \sigma_x}{2}\right) \sin 2\theta + 2\cos 2\theta \left[\frac{\nabla_y - \sigma_x}{2} - \sin^2\theta\right] = 0$

$\therefore \quad (\nabla_y - \sigma_x) \sin 2\theta = 2\tau \cos 2\theta$

$\therefore \quad \tan 2\theta = \frac{2\tau}{\nabla_y - \sigma_x}$,

$\therefore \nabla_{1,} = \left(\frac{\nabla_y - \sigma_x}{2}\right) \sin 2\theta - 2\cos 2\theta \cdot$

hence $\nabla_h = \nabla_x \sin^2\theta + 2\tau_{xy} \cos^2\theta + 2\tau \cos\theta \sin\theta$

$\therefore \nabla_h = \nabla_x \left(\frac{1-\cos 2\theta}{2}\right) + \tau_{xy} \left(\frac{1+\cos 2\theta}{2}\right) + 2\tau \frac{\sin 2\theta}{2}$

$\nabla_h = \frac{\nabla_x + \nabla_y}{2} + \left(\frac{\nabla_y - \sigma_x}{2}\right) \cos 2\theta + 2\tau \sin 2\theta$

$\therefore \tau_{max} = \frac{2\tau}{\nabla_y - \sigma_x}$,

$= (\nabla_y - \sigma_x) \sin\theta \cos\theta - 2(\cos^2\theta - \sin^2\theta)$

$= \nabla_y \cos\theta \sin\theta - \nabla_x \sin\theta \cos\theta + 2\sin^2\theta - 2\cos^2\theta$

$\frac{\partial \varepsilon}{\partial \theta} =$

$\varepsilon_\varepsilon + \varepsilon_x \cdot \frac{P}{Et} \cos\theta + \gamma \cdot \varepsilon \cdot \frac{D}{2} \cos\theta$

$- \varepsilon_y \cdot \frac{\nu}{2} \sin\theta - 2 \cdot \frac{P}{Et} \cos\theta \sin\theta = 0$

$\therefore \varepsilon_\varepsilon = \nabla_y \frac{\varepsilon D}{Et} \sin\theta + 2 \frac{P}{Et} \sin\theta - \nabla_x \frac{PD}{Et} \cos\theta$

$\therefore \varepsilon_\varepsilon E t = \varepsilon_y \cdot \frac{\nu}{2} \sin\theta + 2 \cdot P \sin\theta$

$- \varepsilon_x \cdot P \cos\theta - 2 \varepsilon \cdot \frac{D}{2} \cos\theta$

Sk:

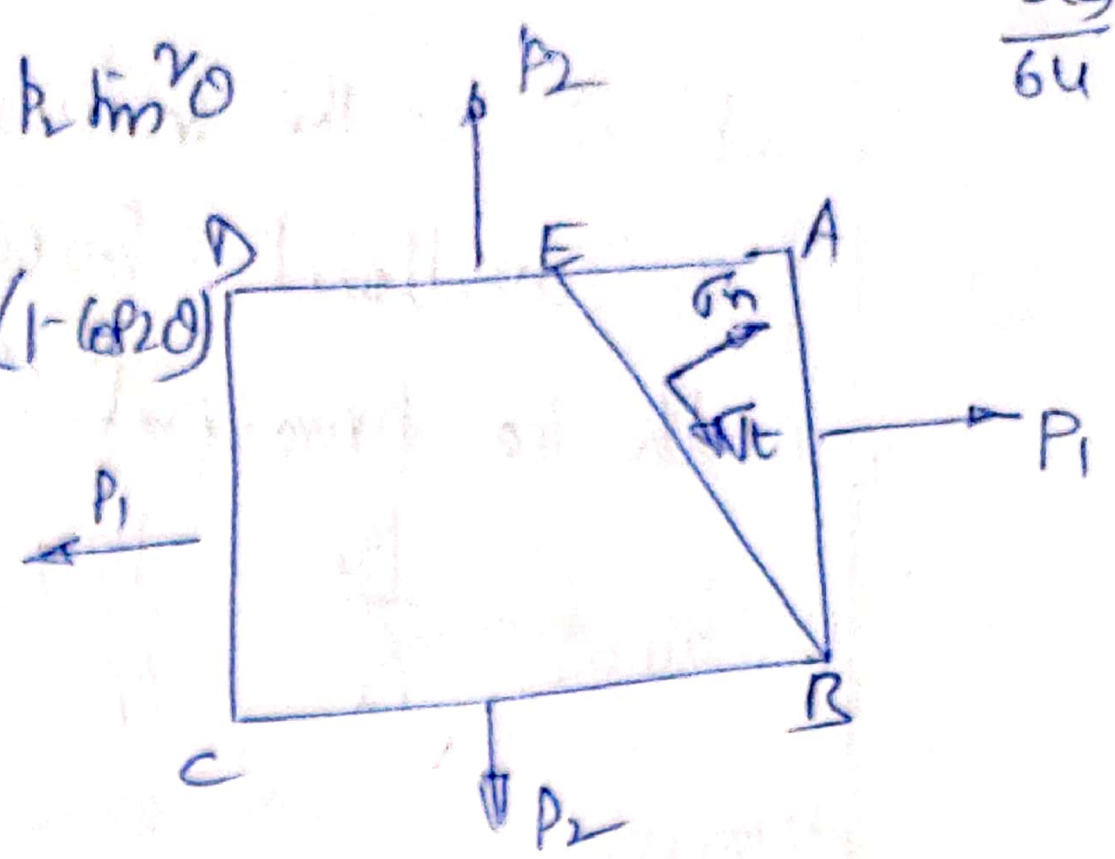
Maximum stress
1-[4]

We have $p_n = p_1 \cos^2\theta + p_2 \sin^2\theta$

$= \frac{p_1}{2}(1+\cos 2\theta) + \frac{p_2}{2}(1-\cos 2\theta)$

$\therefore p_n = \frac{p_1+p_2}{2} + \frac{p_1-p_2}{2}\cos 2\theta$

& $p_t = \left(\frac{p_1-p_2}{2}\right)\sin 2\theta$.



Hence on the plane BE the resultant stress p_r & the resultant of p_n & p_t

$\therefore p_r = \sqrt{(p_1\cos^2\theta + p_2\sin^2\theta)^2 + (p_1-p_2)^2\sin^2 2\theta\cos^2\theta}$

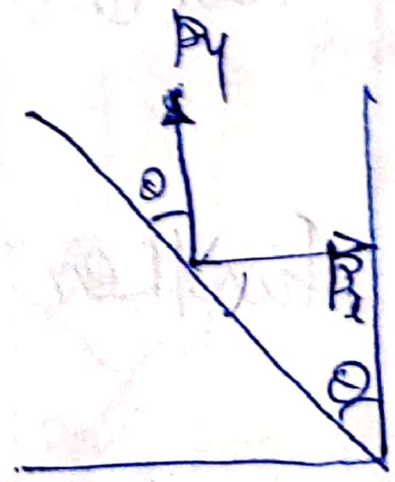
$\therefore p_r = \sqrt{p_1^2\cos^2\theta + p_2^2\sin^2\theta}$

OBLIQUITY: The angle that the resultant stress makes with normal to the plane is called.

If the obliquity be ϕ, we have $\tan\phi = \frac{p_t}{p_n}$

Let p_x be the force on unit area of plane BE ⊥ to the direction of principal stress p_1

Let p_y be " " " p_2



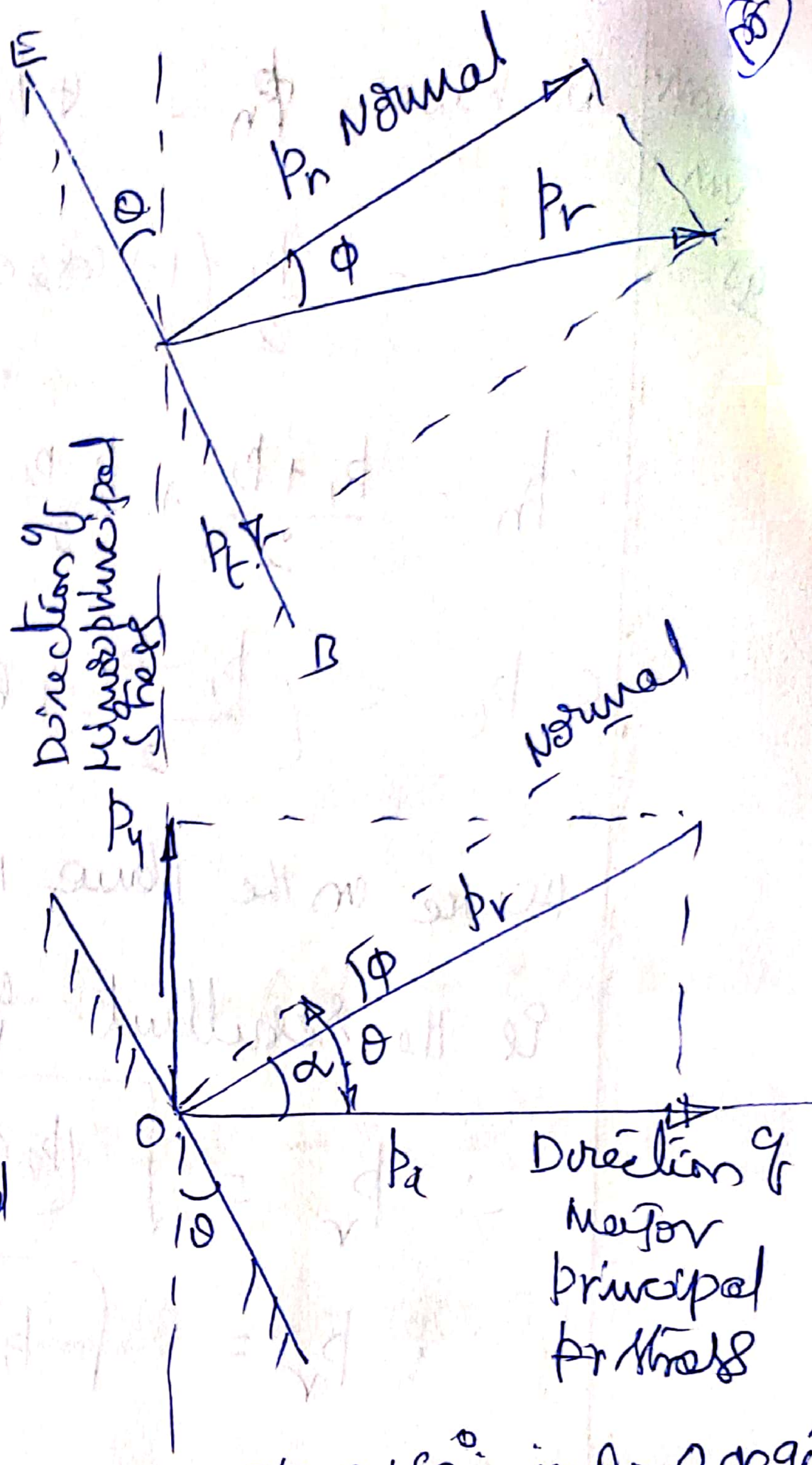
$\therefore$ & $p_x = p_1\cos\theta$, $p_y = p_2\sin\theta$

Resultant stress $p = \sqrt{p_x^2 + p_y^2}$

$= \sqrt{p_1^2\cos^2\theta + p_2^2\sin^2\theta}$ — Result obtained earlier.

Let α be the inclination of the resultant stress with the principal stress p_1

$$\therefore \tan\alpha = \frac{p_y}{p_x} = \frac{p_2}{p_1}\tan\theta$$



Now

$$p_n = \frac{p_1+p_2}{2} + \frac{p_1-p_2}{2}\cos 2\theta$$

$$p_t = \frac{p_1-p_2}{2}\sin 2\theta.$$

For the principal planes p_t (shear stress) should be equal to zero.

ie $\frac{p_1-p_2}{2}\sin 2\theta = 0 \Rightarrow 2\theta = 0 (or) 180^\circ$ (ie) $\theta = 0$ or 90°

when $\theta = 0$, $p_n = \frac{p_1+p_2}{2} + \frac{p_1-p_2}{2}(1) = p_1$

when $\theta = 90^\circ$, $p_n = \frac{p_1+p_2}{2} + \frac{p_1-p_2}{2}(-1) = p_2$

Further p_t is max when

$\sin 2\theta = 1$

$2\theta = 90^\circ (or) 270^\circ$ (or) $\theta = 45^\circ (or) 135^\circ$

$\therefore$ Max shear stress $p_{t(max)} = \frac{p_1-p_2}{2}$

Note: There are two mutually $\perp$r planes along with the greatest shear stress occurs. These planes are at angles of 45° & 135° with principal plane along which principal stress p_1

A round bar of 60mm dia & 6m length carries a load of 280kN and extends 1.6 mm. Estimate max shear stress in the bar & the value of modulus of elasticity.

Sol: Area of CS of bar $= \frac{\pi}{4}(60)^2 = 1024\ mm^2$.

Total load carried $= 280\ kN$

Direct stress in the bar $\sigma = \frac{280\times1000}{3024\times10^{-6}} = 55.75\ MN/m^2$

Max shear stress $\tau_{max} = \frac{\sigma}{2} = \frac{1}{2}$ Direct stress.

$= \frac{55.75}{2} = 27.875\ MN/m^2$

Strain $e = \frac{1.6}{6\times1000} = 0.000266$ $\left[e = \frac{\Delta L}{L}\right]$

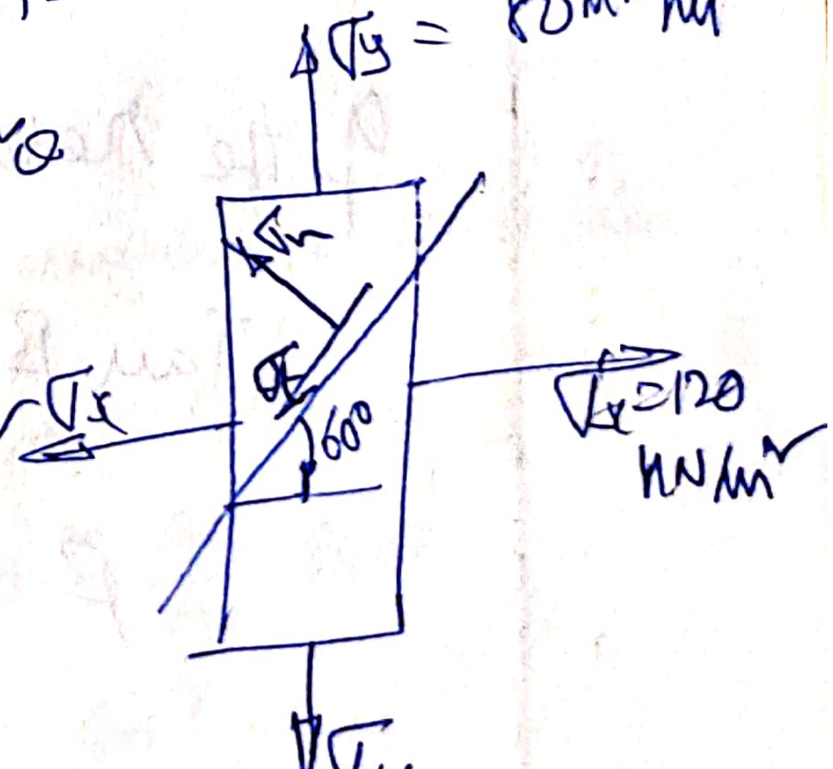
$\therefore$ Youngs modulus $E = \frac{\text{Stress}}{\text{Strain}} = \frac{55.75\times10^6}{0.000266} = 209.6\times10^3\ N/mm^2$

(PP) A steel plate is subjected to tensile stresses of 120 MN/m² and 80 MN/m² at right angles to one another. Determine the normal & tangential stress on a plane inclined @ 60° to 120 MN/m².

Sol: Normal stress $\sigma_n = \sigma_y\cos^2\theta + \sigma_x\sin^2\theta$

$\sigma_n = 80\cos^2 60 + 120\sin^2 60$

$= 20 + 90 = 110\ MN/m^2$

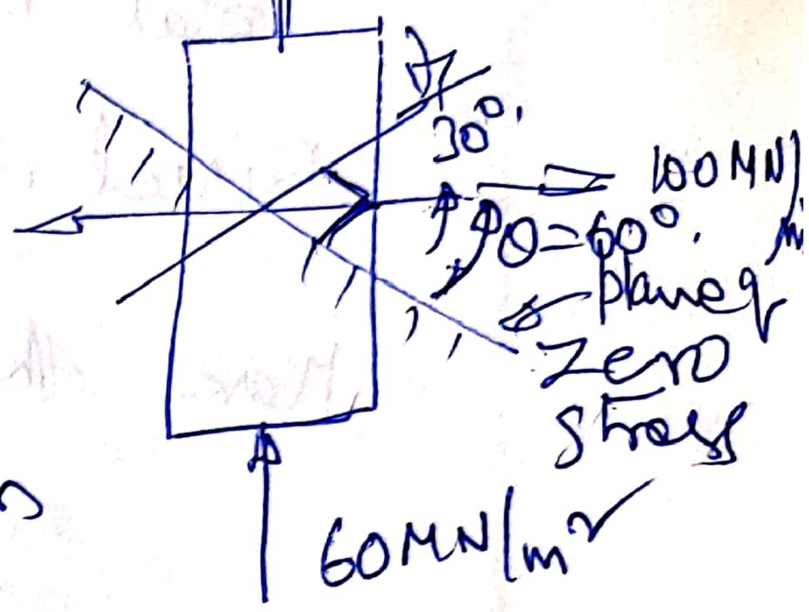


Tangential stress

$\sigma_t = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$

$\sigma_t = \left(\frac{120 - 80}{2}\right)\sin 120^\circ = 20\times0.866 = 17.32\ MN/m^2$

(10). The direct stress at a point in a piece of ma are 100 MN/m² tensile, 60 MN/m² Compressive. Fi the intensity and direction of stress across a plane, the normal of which is inclined @ 30° to the axis of 100 MN/m² direct stress, this plane being 1r to the plane of Zero stress [FG]



Sol: Since the normal to the plane is inclined at an angle 30° to the 100 MN/m² stress direction

$\theta = 60°$.

$\sigma_x = 100 \times 10^6$ N/m², $\sigma_y = -60 \times 10^6$ N/m²

$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta = 60$ MN/m²

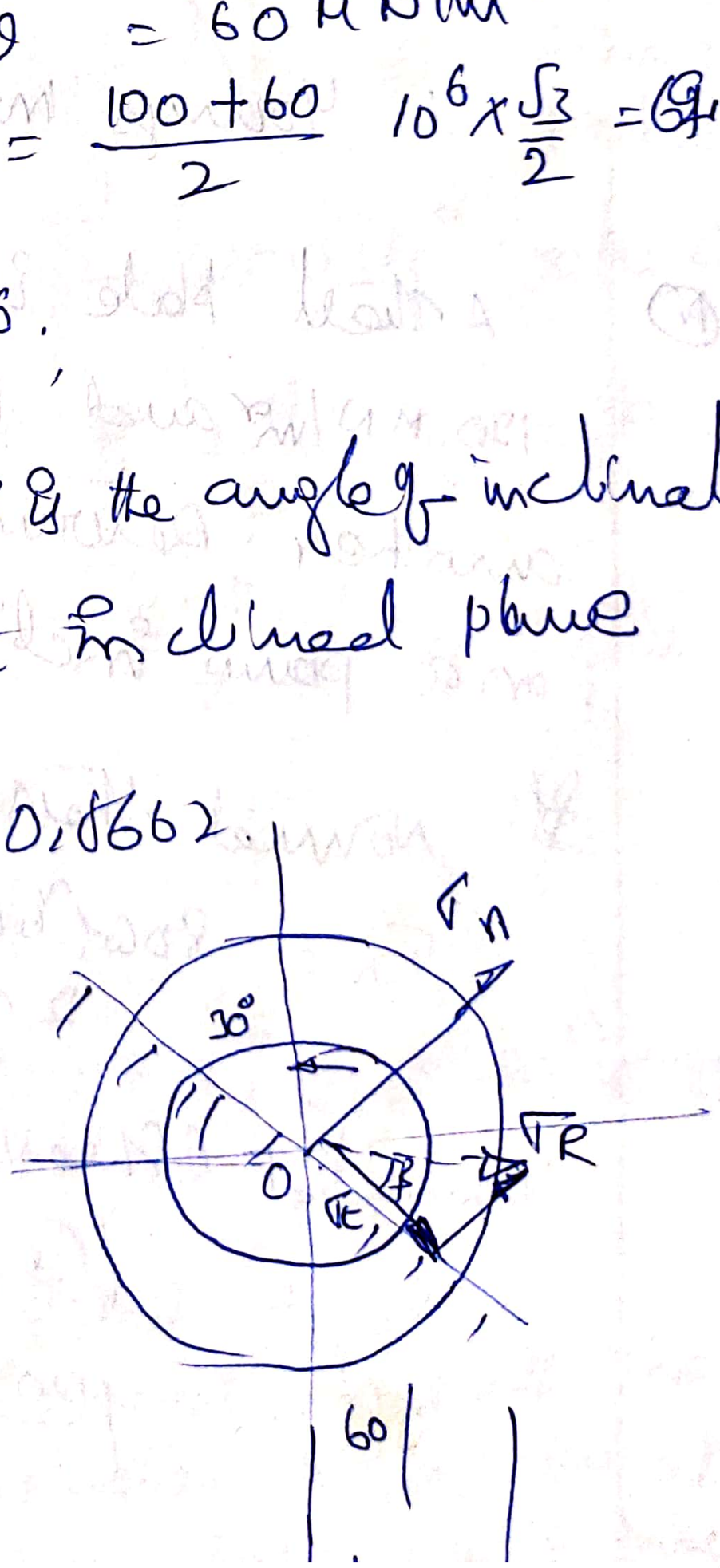
$\sigma_t = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta = \frac{100+60}{2} \times 10^6 \times \frac{\sqrt{3}}{2} = 69.2$ MN/m²

$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = 91.66$.

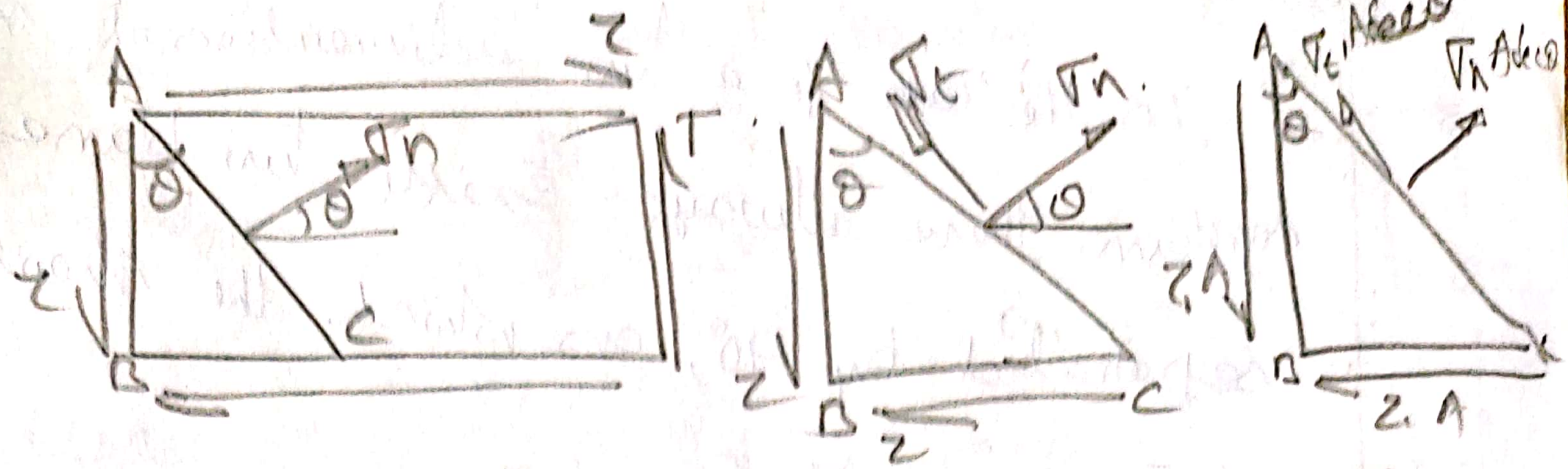
$\tan\beta = \frac{\sigma_n}{\sigma_t}$ where β is the angle of inclination of the resultant σ_R to the inclined plane

$\tan\beta = \frac{60}{69.28} = 0.8662$.

$\beta = 40°54'$



STRESSES INDUCED BY STATE OF SIMPLE SHEAR



(a) (b) Stresses (c) Forces

Fig(a) shows the state of simple shear or pure shear. b & c show wedge shaped stress element. It is required to find stresses τ_n & τ_t on inclined plane. Let Area of AB = A Then Area of AC = A Sec θ

Considering unit thickness of the element and balancing forces in direction normal to AC.,

$$\tau_n \cdot AC = \tau AB \sin\theta + \tau BC \cos\theta.$$

$$\tau_n = \tau \frac{AB}{AC} \sin\theta + \tau \frac{BC}{AC} \cos\theta.$$

$$\therefore \tau_n = \tau \cos\theta \sin\theta + \tau \sin\theta \cos\theta = 2\tau \sin\theta \cos\theta$$

$$\tau_n = \tau \sin 2\theta \quad -(1)$$

Similarly resolving Forces along Tangential direction

$$\tau_t AC = -\tau BC \sin\theta + \tau AB \cos\theta$$

$$\tau_t = -\tau \frac{BC}{AC} \sin\theta + \tau \frac{AB}{AC} \cos\theta$$

$$\tau_t = -\tau \sin^2\theta + \tau \cos^2\theta$$

$$\tau_t = \tau (\cos^2\theta - \sin^2\theta) = \tau \cos 2\theta.$$

From (1) $|\tau_n|_{max} = \tau$ when $\theta = 45^\circ$ & $\tau_t = 0$.

Principal Stresses and principal planes.

In the case of a two-dimensional stress system there always exists two planes separated by 90°, on which the shearing stresses are zero.

These 'shear-free' planes are called principal planes and the corresponding values of normal stresses σ_n acting on these planes are called principal stresses.

The general expression for σ_n on any plane inclined at an angle θ is

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\theta + \tau \sin 2\theta$$

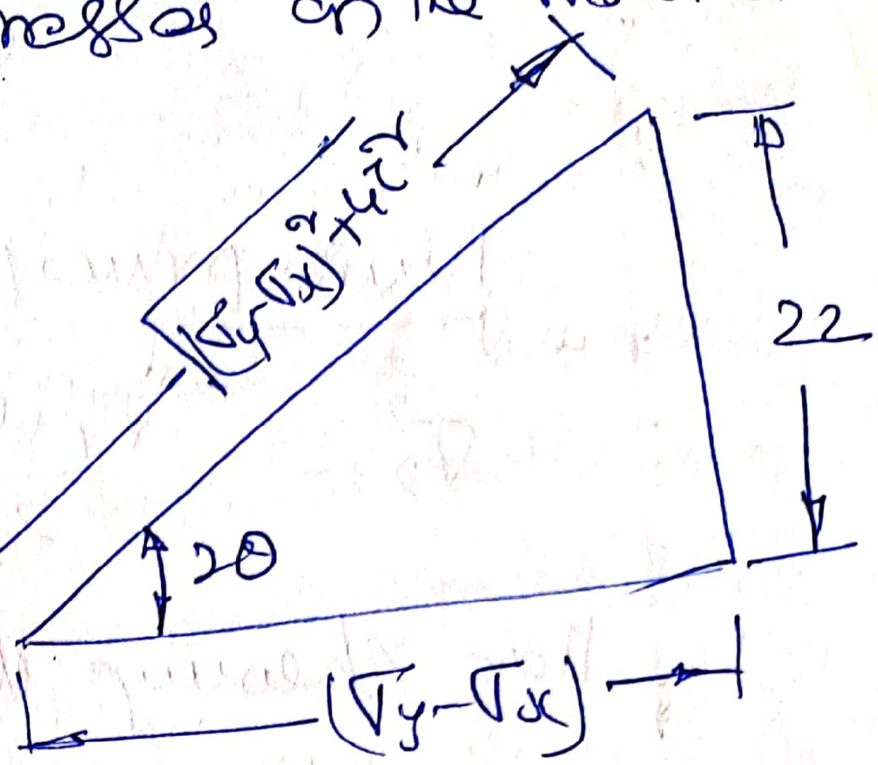
$\therefore$ For $(\sigma_n)_{max}$, $\tan 2\theta = \dfrac{2\tau}{\sigma_y - \sigma_x}$

which defines the directions of the planes on which the principal stresses act.

The principal stresses are nothing but max and minimum direct stresses on the material

$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

$$\cos 2\theta = \pm \frac{\sigma_y - \sigma_x}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$



Note: The greatest normal stress i.e the max principal stress which shall not exceed a permissible value for the safety of the design is to be found out.

The magnitudes of the principal stresses can be got by substituting the values of θ in equation

If σ_1 & σ_2 are the principal stresses then

$$\sigma_1 \text{ or } \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \frac{\sigma_y - \sigma_x}{2} \frac{(\sigma_y - \sigma_x)}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}} + \tau \frac{2\tau}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

$$= \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \left[\frac{(\sigma_y - \sigma_x)^2 + 4\tau^2}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}} \right]$$

$$= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau^2}$$

∴ Major principal stress

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau^2}$$

Minor principal stress:

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_y-\sigma_x}{2}\right)^2+\tau^2}$$

Max shearing stress $\tau_{max} = \left(\frac{\sigma_y-\sigma_x}{2}\right)\sin 2\theta - \tau\cos 2\theta$ —(1)

To find the planes on which the shear stress attains its max value $\frac{d\tau}{d\theta}=0$

i.e $\frac{\sigma_y-\sigma_x}{2}(\cos 2\theta_s) + 2\tau\sin 2\theta_s = 0$

$\therefore \tan 2\theta_s = -\left(\frac{\sigma_y-\sigma_x}{2\tau}\right)$

Therefore, the planes of max. shear stress are inclined @ 45^0 to the principal planes.

Sub. θ_s in eq(1) ie $\tau_\theta = \left(\frac{\sigma_y-\sigma_x}{2}\right)\sin 2\theta - \tau\cos 2\theta$

$$(\tau_\theta)_{max} = \pm\frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}$$

$$= \frac{1}{2}[\text{Difference of principal stresses}]$$

$$\text{i.e } (\tau_\theta)_{max} = \frac{1}{2}(\sigma_1-\sigma_2)$$

and each value has same magnitude, the direction being reversed when θ_s is increased by 2 ± 90^0

MOHR'S CIRCLE OF STRESS

Mohr's Circle diagram is a graphical representation of the stresses in an element subjected to a general two-dimensional stress system.

By this method, the principal stresses, the maximum shear stress and the normal and tangential stresses on a given plane may be found by a simple graphical Con

Mohr Circle represents normal & shear stress components graphically in a manner that makes them more easy to understand and remember.

normal stress $\sigma_n = \left[\frac{\sigma_x+\sigma_y}{2}\right] + \left[\frac{\sigma_y-\sigma_x}{2}\right]\cos 2\theta + \tau \sin 2\theta$

~~Shear~~ Shear stress $\sigma_t = \left[\frac{\sigma_y-\sigma_x}{2}\right]\sin 2\theta - \tau \sin \cos 2\theta$.

Squaring both sides & each eqn's and adding

$$\left[\sigma_n - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right]^2 + (\sigma_t)^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2$$

This is the eqn's of a circle of the form

$$(\sigma_n - a)^2 + (\sigma_t)^2 = R^2$$

where the radius $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2}$

and $a = \frac{\sigma_x+\sigma_y}{2}$.

The centre of Circle lies @ pt $\left[\frac{\sigma_x+\sigma_y}{2}, 0\right]$.

Note: The centre of the circle always lies on the σ_x axis

Basic steps for drawing Mohr's circle of stress:

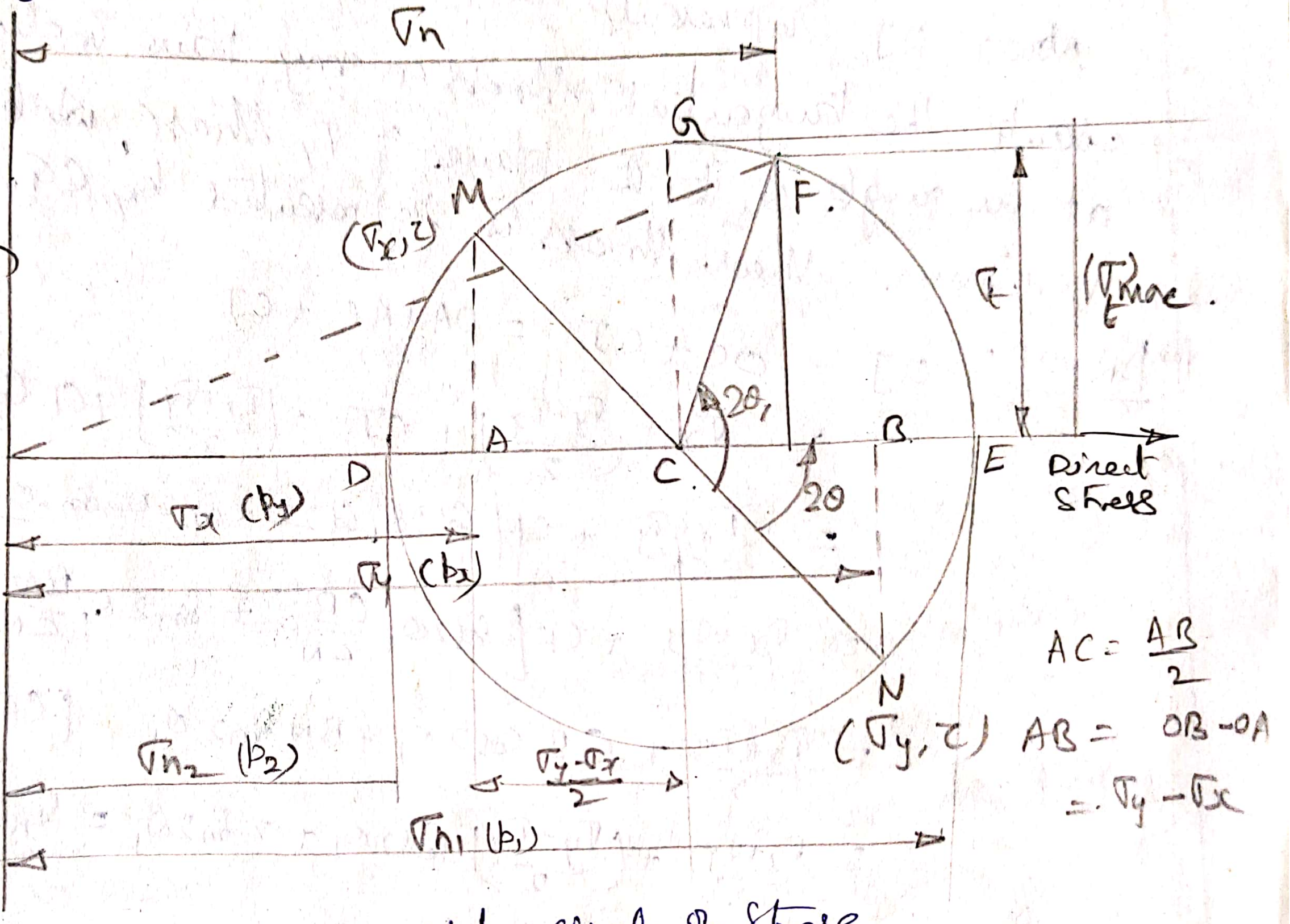
1. The normal stresses σ_x & σ_y are plotted along the abscissa. The tensile stresses are considered as +ve (plotted to the right of origin). The compressive stresses are considered -ve (plotted to the left of the origin.)
2. The shearing stresses τ are plotted as ordinates positive shearing stresses [clockwise rotation of the element ($\downarrow \square \rightarrow$)] are plotted above abscissa, the -ve shear stress ($\uparrow \square \downarrow$) are plotted below the abscissa.
3. +ve angles on the circle are obtained when measured in the counter-clockwise sense and -ve angles on the circle are obtained in the clockwise sense. An angle of 2θ on the circle corresponds to an angle of θ on the element.

Note: The plane in the given problem is represented by a point on the Mohr's Circle.

The stresses acting on the plane become coordinates of the point in the Mohr's circle.

procedure to Construct Mohr's Circle:

If the material is subjected to a general two-dimensional stress system consisting of two normal stresses σ_x and σ_y along with a state of simple shear, the procedure to construct the Mohr circle stress is



Mohr Circle for Stress

Construction: Along the x axis, OA and OB are set off equal in length to σ_x & σ_y respectively

AM & BN are drawn @ A & B ⊥r to x axis each equal to CM as radius, a circle is drawn. Let the angle BCN = 2θ.

Now OE & OD represent the maximum and minimum principal stress respectively and tan 2θ gives the direction of planes on which these principal stresses act.

We now find the normal and tangential stresses on a plane inclined @ θ_1 to the plane of τ_y stress.

From CN, angle $2\theta_1$ is measured in counter clockwise direction and the point F is obtained on the circle

FJ is drawn ⊥r to OE

Now OJ represents the normal stress & FJ represents the tangential stress on any plane inclined at an angle θ_1 to the plane of τ_y stress, while the maximum shear stress is represented by CG.

PROOF: $OJ = OC + CJ = OA + AC + CJ$

$= \tau_x + \left(\frac{\tau_y - \tau_x}{2}\right) + CJ = \left(\frac{\tau_x + \tau_y}{2}\right) + CF\cos(2\theta_1 - 2\theta)$

$= \frac{\tau_x + \tau_y}{2} + CF[\cos 2\theta_1 \cos 2\theta + \sin 2\theta_1 \sin 2\theta]$

$= \frac{\tau_x + \tau_y}{2} + CF\left[\cos 2\theta_1 \frac{CB}{CN} + \sin 2\theta_1 \frac{BN}{CN}\right]$

$= \frac{\tau_x + \tau_y}{2} + CB\cos 2\theta_1 + BN \sin 2\theta_1 \quad [\because CF = CN]$

$= \frac{\tau_x + \tau_y}{2} + \left(\frac{\tau_y - \tau_x}{2}\right)\cos 2\theta_1 + \tau \sin 2\theta_1 = \tau_n$

Tangential stress FJ $= CF\sin(2\theta_1 - 2\theta)$

$= CF[\sin 2\theta_1 \cos 2\theta - \cos 2\theta_1 \sin 2\theta]$

$= CF\left[\sin 2\theta_1 \frac{CB}{CN} - \cos 2\theta_1 \frac{BN}{CN}\right]$

$= CB\sin 2\theta_1 - BN\cos 2\theta_1$

$= \left(\frac{\tau_y - \tau_x}{2}\right)\sin 2\theta_1 - \tau\cos 2\theta_1 = \tau_t.$

Max shear stress is represented by CG

i.e $[\tau_t]_{max} = CG = CN = \sqrt{CB^2 + BN^2}$

$= \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}$

$= (\tau_t)_{max}$.

This $(\tau_t)_{max}$ is also equal to $\frac{\sigma_{n_1} - \sigma_{n_2}}{2}$

Principal stresses are represented by OE & OD.

Major principal stress : $= OE$

$OE = OC + CE = OC + CN = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}$

$\therefore OE = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2} = \sigma_{n_1}$ = Major principal stress

Minor principal stress $= OD = OC - CD = OC - CN$

$= \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$

$= \sigma_{n_2}$ = Minimum (minor) principal stress.

To find the directions of principal planes:

We know that principal planes are 'shear-free' planes

i.e on the principal planes, shear $\tau_t = 0$.

$\therefore \tau_t = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta = 0$

$\therefore \tan 2\theta = \frac{2\tau_{xy}}{(\sigma_y - \sigma_x)}$ in the Mohr's Circle when τ_t is made equal to zero, i.e 2θ, coincides with 2θ $\tan 2\theta_1 = \frac{2\tau}{\sigma_y - \sigma_x}$

$\therefore \theta$, & $(90 + \theta)$ are the value of angles which give the orientation of principal planes.

MECHANICS OF SOLIDS

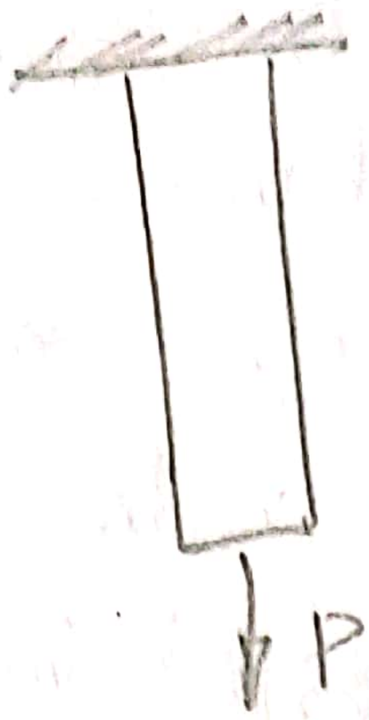
UNIT-I : SIMPLE STRESSES & STRAINS: Elasticity & plasticity - Types of Stresses & Strains - Hooke's Law - Stress-Strain diagram for Mild Steel - Working Stresses - Factor of Safety - Lateral Strain - Poisson's ratio & Volumetric Strain. Elastic modulii & relationship between them - Bars of Varying Section - Composite bars - Temp Stresses, Strain energy - Resilience - Gradual, Sudden, impact & Shock loadings.

Mechanics of Solids [or] Strength of materials] helps determine the stresses and deflections in structures members under various load combinations & predict their behaviour.

A structure designed to carry loads Comprises various members such as beams, Columns and Rods.

Strength of materials is a subject which deals with the load carrying capacity [strength] rigidity & stability of isolated structural members such as beams & Columns.

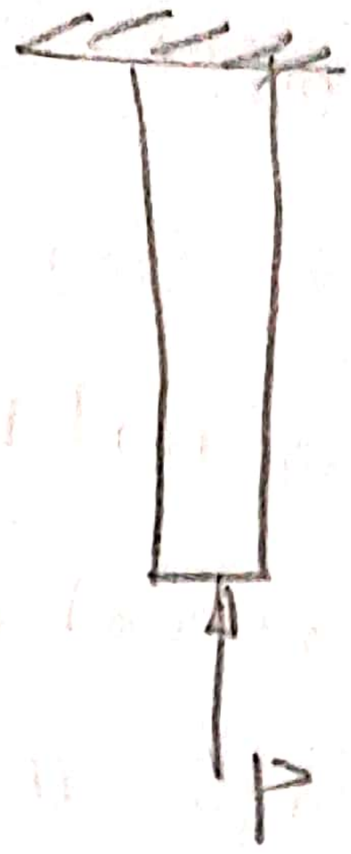
FORCES (OR) LOADS



(a) Tension

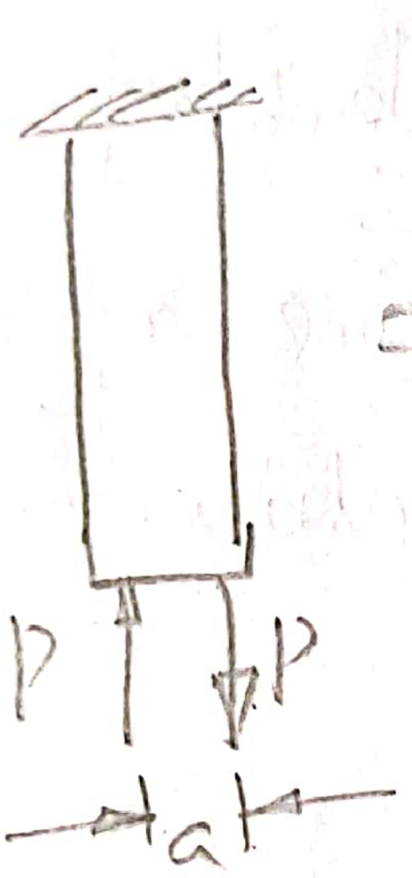
Ex: A rope lifting loads
(a) pulling a wagon





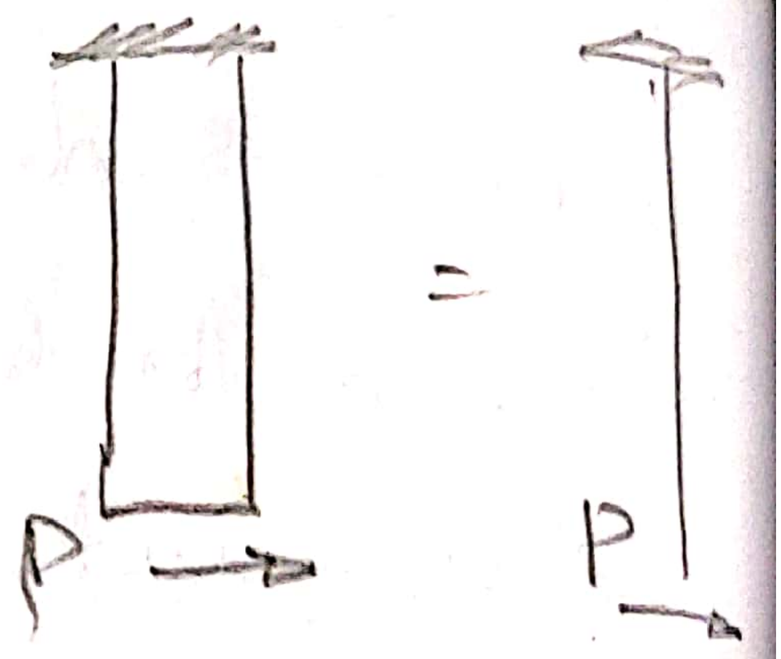
(b) Compression

Ex: Walls, structure
Columns & legs





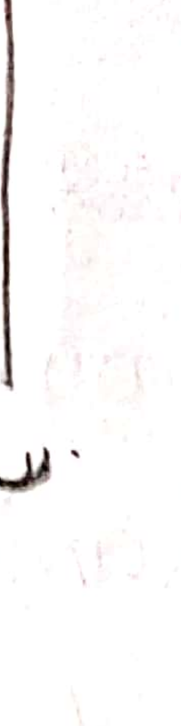




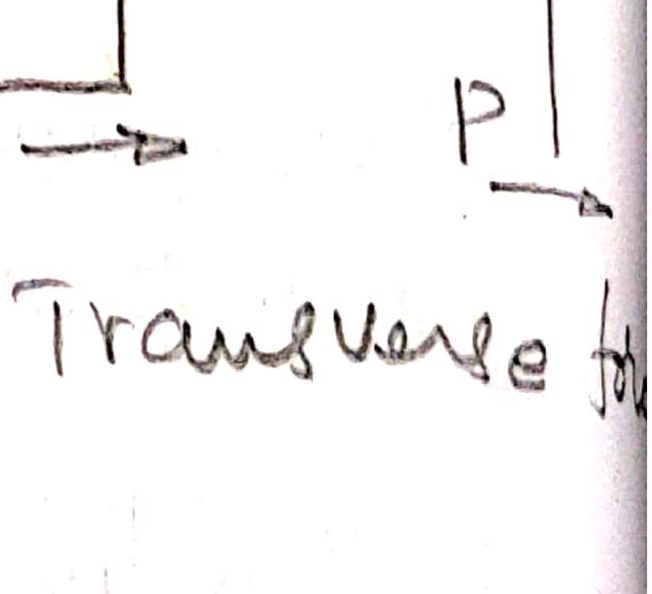
(c) Moment

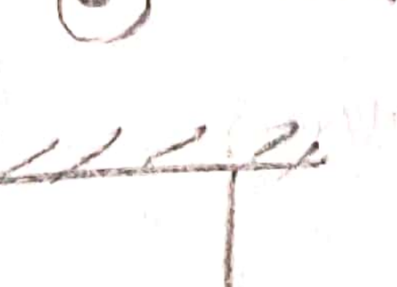
(d) Transverse fo





$M_t = Pa$





Twisting Moment

P

(e) Torsion

(f) Transverse fo
on a beam

EXTERNAL FORCES

forces acting along the axis of a body are known as direct forces (or) axial forces [A.F]. Two types of A.F

1) Axial tensile force (a pull on a body)
2) Axial Compressive force [a push on a body]

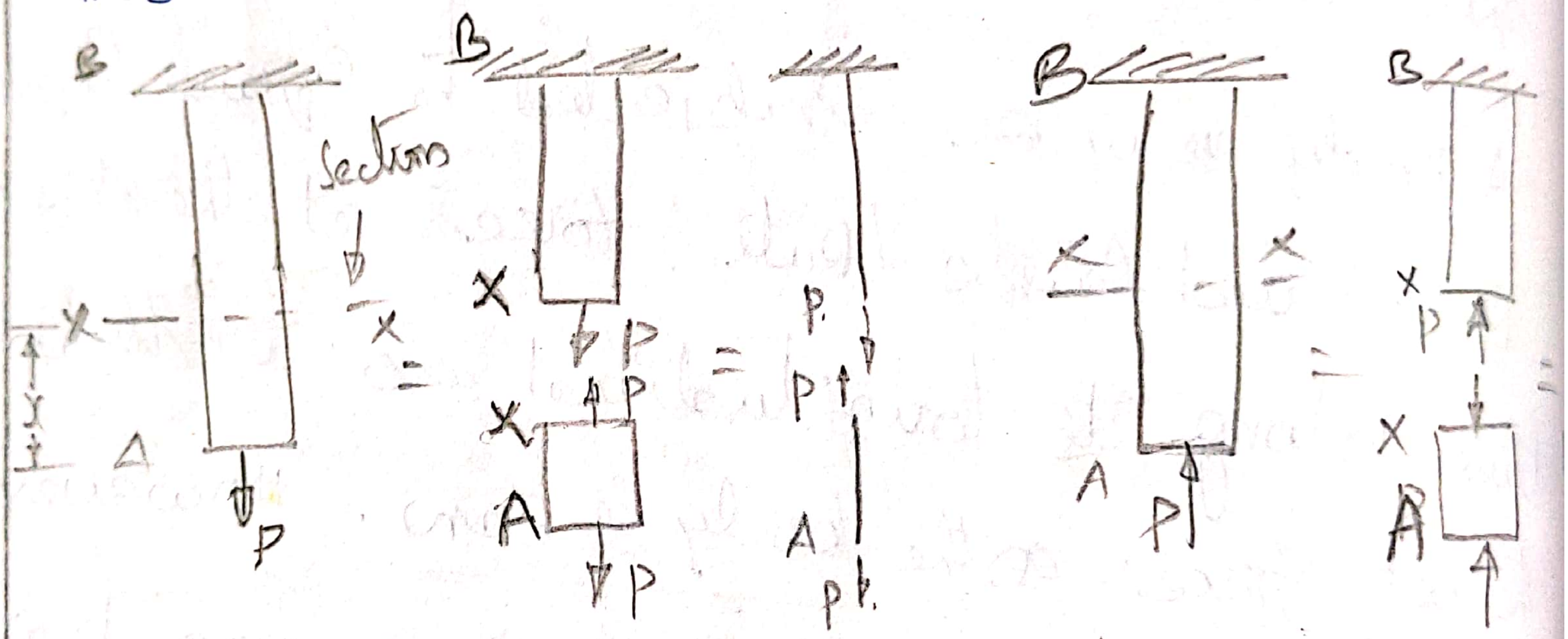
Axial forces (or loads) acting on a body are indicated above. Fig(a) indicates tensile force on a body - the force pulls the body and causes it to elongate.

Fig(b) the force compresses [pushes] the body and causes it to shorten in length

A body may be subjected to equal Compressive [push] and tensile [pull] forces at the same line along its longitudinal axis as shown in (c) the net force on the body is zero. However, these force causes Compression in one part of the body & tension in the other part. As a result, a part of the body shortens & other part elongates. The forces form a couple of magnitude P a (force x distance), where P is the magnitude of the forces and a is the distance between them. Such a force system is a special case of axial loading and is known as bending moment

Transverse forces: External forces acting on the axis of a body are known as transverse forces. These forces tend to bend the body as well as shear (tear) it off.

Internal forces of a body: External forces [or applied] on a body are transmitted to all parts through the material of the body. Such a transmission of forces through a body causes the body to deform and develop equal and opposite internal resisting forces. These are known as internal forces.



(a) Tension FBD (b) Compression

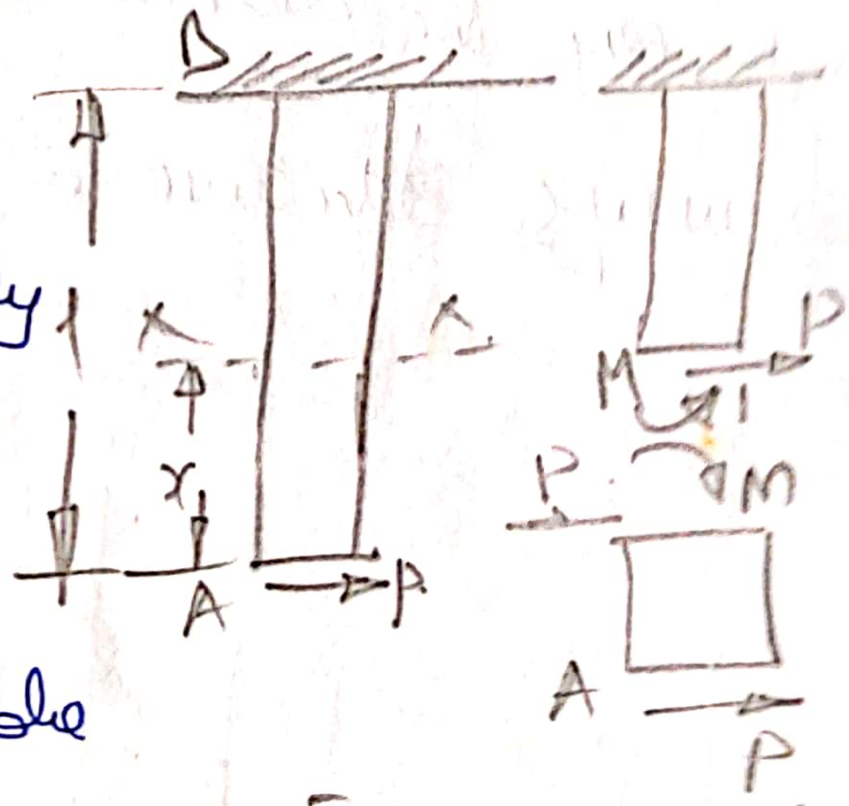
Fig (a) indicates a body subjected to an external pull at its free [un supported] end. Consider an arbitrary section x-x of the body at a distance x from its free end. Segment B of the body would develop a resisting force of magnitude P@x acting in the direction opposite to that of P, so as to maintain the equilibrium of segment A. Similarly, segment B of the body would develop a resisting force of magnitude P@x acting in the direction opposite to that of P, so as to maintain the equilibrium of segment A. Such forces are known as internal forces.

If the part of body below section x-x is separated there would be a force of magnitude equal to applied P but in opposite direction at section x-x. The force at section x-x is required to maintain the equilibrium of segment A. Similarly, segment B of the body would develop a resisting force of magnitude P@x acting in the direction opposite to that of P, so as to maintain the equilibrium of segment A. Such forces are known as internal forces.

The forces @ section x-x of both segments are external to the body, and are evident only when the segments are notionally separated out. These forces are known as internal forces of the body at the section.

In this case, the internal force P & the distance x of the section from free end, the section is chosen arbitrarily, it can be found (inferred) that the internal force P is constant at any section in the body.

Similar Conclusion can be drawn for the body of a Compressive force, to an end moment (BM)



Transverse force

However, the transverse force P acting on the body induces a force of magn P to the left as well as C.W moment of magnitude M (= Px) at section x-x of segment A x of the body.

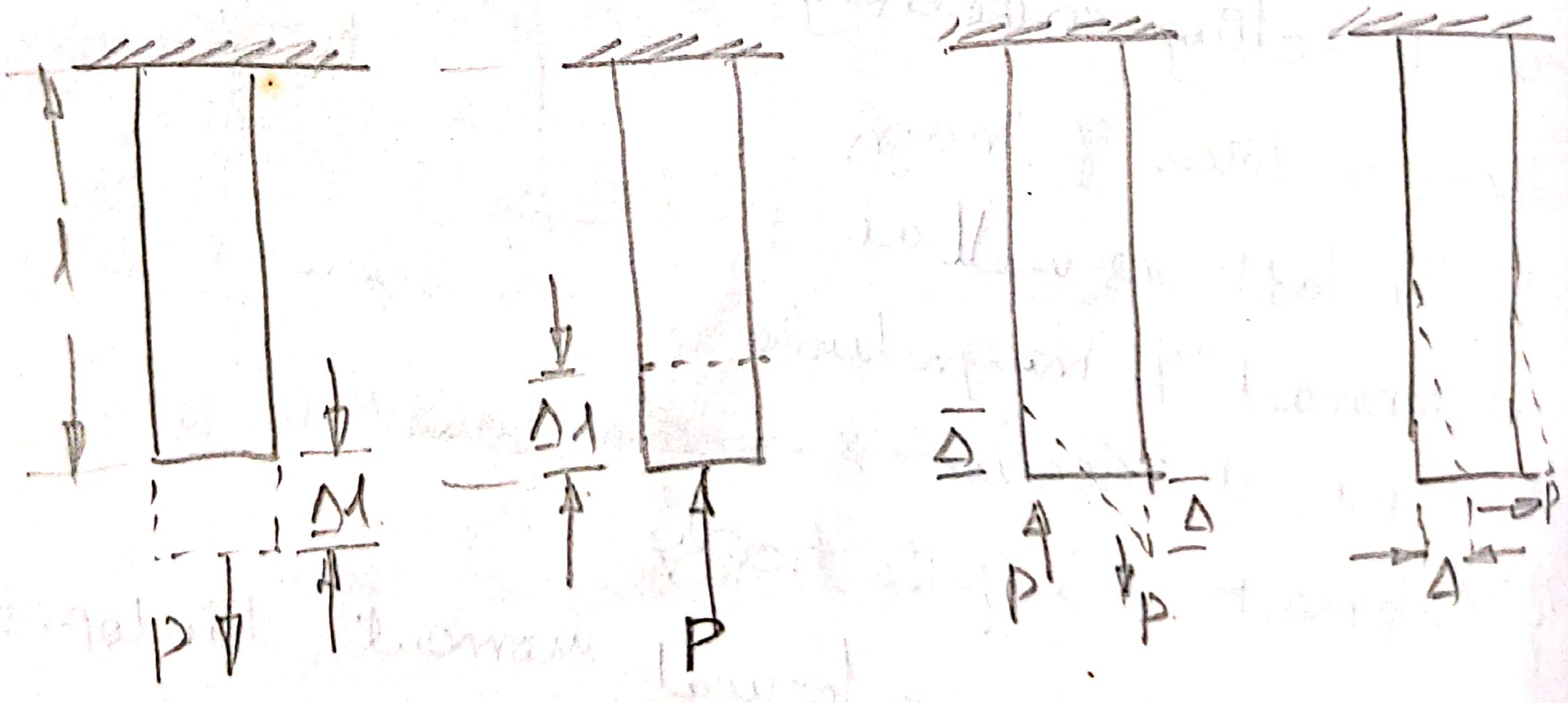
In this case, the internal moment developed is not constant but is a function of the distance of the section in the body from the free end; the magn of M varies from zero at A (free end) to Pl at B (fixed end). It is obvious that the internal forces may not be constant in a body, and may vary from

Deformations of bodies [Deformation: change and size] :

External loads applied on a body are transmitted to its supports through its material as explained earlier.

In such a process, the body is S.T deformed depending upon the applied load & supports.

In the elementary analysis, the deformations of the bodies are assumed to be very small compared to their dimensions. The geometry of the body & portions of loads are assumed to remain unaltered by the deformations. However the deformations are always drawn to an exaggerated scale.



a) Elongation (b) Shortening c) Bending. d) Transverse

Deformations of typical structural members

Elasticity & Materials: In the elementary theory & analysis the material is assumed to be perfectly elastic. i.e the deformations & a body due to applied loads disappear as soon as the loads are removed.

The molecules & a material are held together by inter molecular bonds and resist any change in their alignment. When an external load is applied the particles & the body are displaced which is resisted by the internal forces.

Equilibrium & the forces is reached when the external forces are balanced by the internal forces.

The displacements & the molecular particles result in the deformations (change in the shape & size) & a body.

If the body is perfectly elastic, its original shape is regained when the external forces are removed.

The body is termed plastic, if the deformed body shape persists even after the removal & external forces.

Elasto - plastic If the body regains its original shape & size only partly, it is known

Elastic Limit: The load upto which a materia
elastic & known as elastic limit & mate
Ex: Concrete & Metal alloys are elastic
plastic materials: clay & plasticine

Other assumptions: The materials are also assu
be isotropic [have same properties in a
and homogeneous [have " " anywhere

Some materials are orthotropic [have diff
properties in orthogonal directions)
Ex: Timber & composites.

Materials with different properties in dif
directions are known as Isotropic materi

DIRECT STRESSES

Stress: It is a common observation that larger sized bodies can sustain greater loads than smaller bodies of the same material.

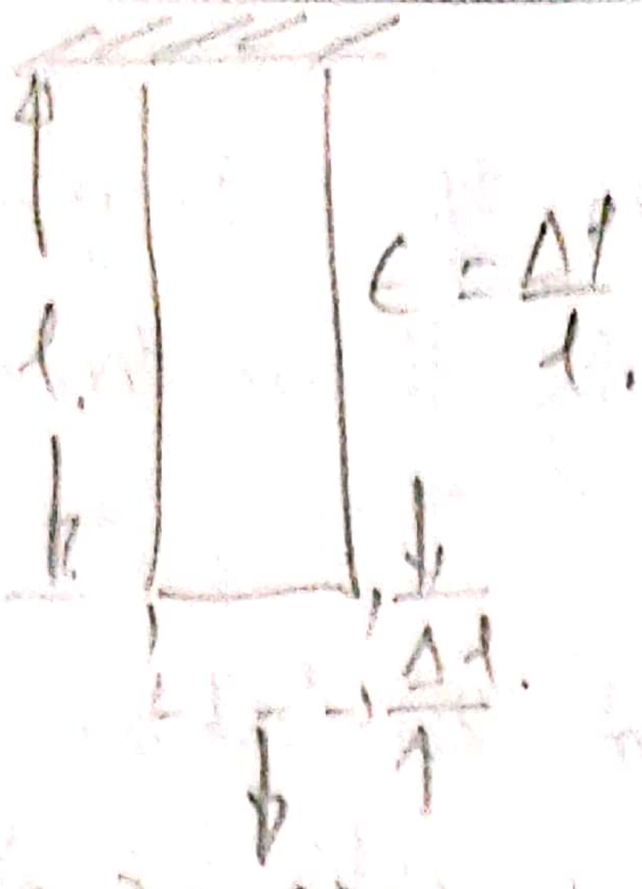
Thus, the load alone cannot be taken as the criterion to describe the capacity (or strength) of a material. Since load sustained by a body is proportional to the area over which the load is acting, the load per unit area provides a better concept of material capacity than load alone. It also helps to compare the load carrying capacities of various materials.

The parameter, load per unit area is known as intensity of stress (or) simply stress.

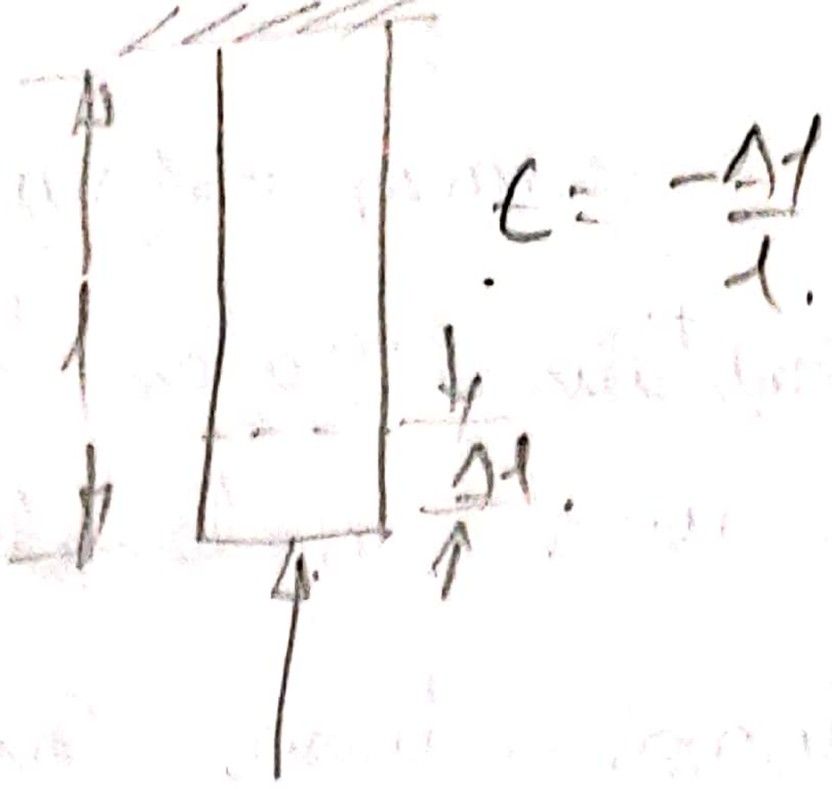
Avg intensity of stress $\sigma = \frac{P}{A}$

Where σ = stress
(+ve for tensile
-ve for compr
P = load app
A = Area of

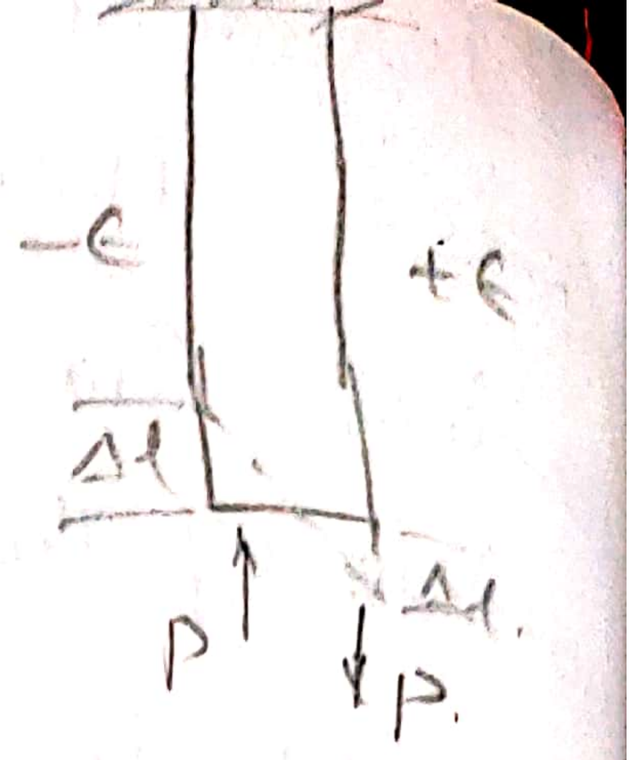
However if the stress is not uniform over the section the avg intensity of stress at any point



(a) Tension



(b) Comp



(c) Bending

Strain: The deformation of a body sub. to load is proportion to its length.

The deformation per unit length is known as

$$\epsilon = \Delta l / l.$$

where l = original length of body

Δl = Change in the length [Elongation & Shortening]

The strain obtained from above eqn is avg value over length l.

However if strain is not uniform over the length of a body, strain at a point [local strain] is defined over an elemental length dx.

The strain at a point is defined as

$$\epsilon = \frac{\Delta dx}{dx}.$$

where dx = original length of the body

Δdx = Change in the length.

NOTE: If The change in the length of a body cannot be seen by naked eye for common construction materials as steel & concrete.

The change in the length of the body is +ve if the body changes elongates [+ve Strain]; the change is -ve, if the length decreases (-ve Strain). Accordingly, the strain is +ve if the body elongates & -ve if it shortens

A body, when subjected to bending, shortens on one side & elongates on the other side of the body with Zero Strain at the centroid.

NOTE The change in the dimension of a body may be due to applied loads (or) due to other causes such as temperature changes.

Thermal Strain: The Strain due to temperature changes is known.

Hooke's law & Young's modulus: within the limit of elasticity of a material, Strain is proportional to Stress applied.

A material is said to be elastic if all deformations [Strains] vanish when the applied stress is removed.

By Hooke's law $\epsilon \propto \sigma$ (or) $\sigma = \sigma \propto \epsilon$

$$\therefore \sigma = \epsilon E \Rightarrow E = \frac{\sigma}{\epsilon}.$$

where E = proportionality Constant

'E' is known as young's modulus (or) modulus of elasticity.

The values of Constant of proportionality for Hoo
had been determined by Thomas young for
material within elastic limit. [Ie Hooke
Recognise a limit to this proportionality]

Young had found that the proportionality
Constant has a unique value for a material
Its elastic limit.

∴ Young's modulus is thus the stress requi
to Cause unit strain in a material.

Hard materials have high values for young
Diamond is the substance with highest va
of young's modulus

NOTE: The value of young's modulus is une
the same in Compression as tension for m
of metals & alloys.

Some materials such as Concrete &
display different characteristics in compr
& tension.

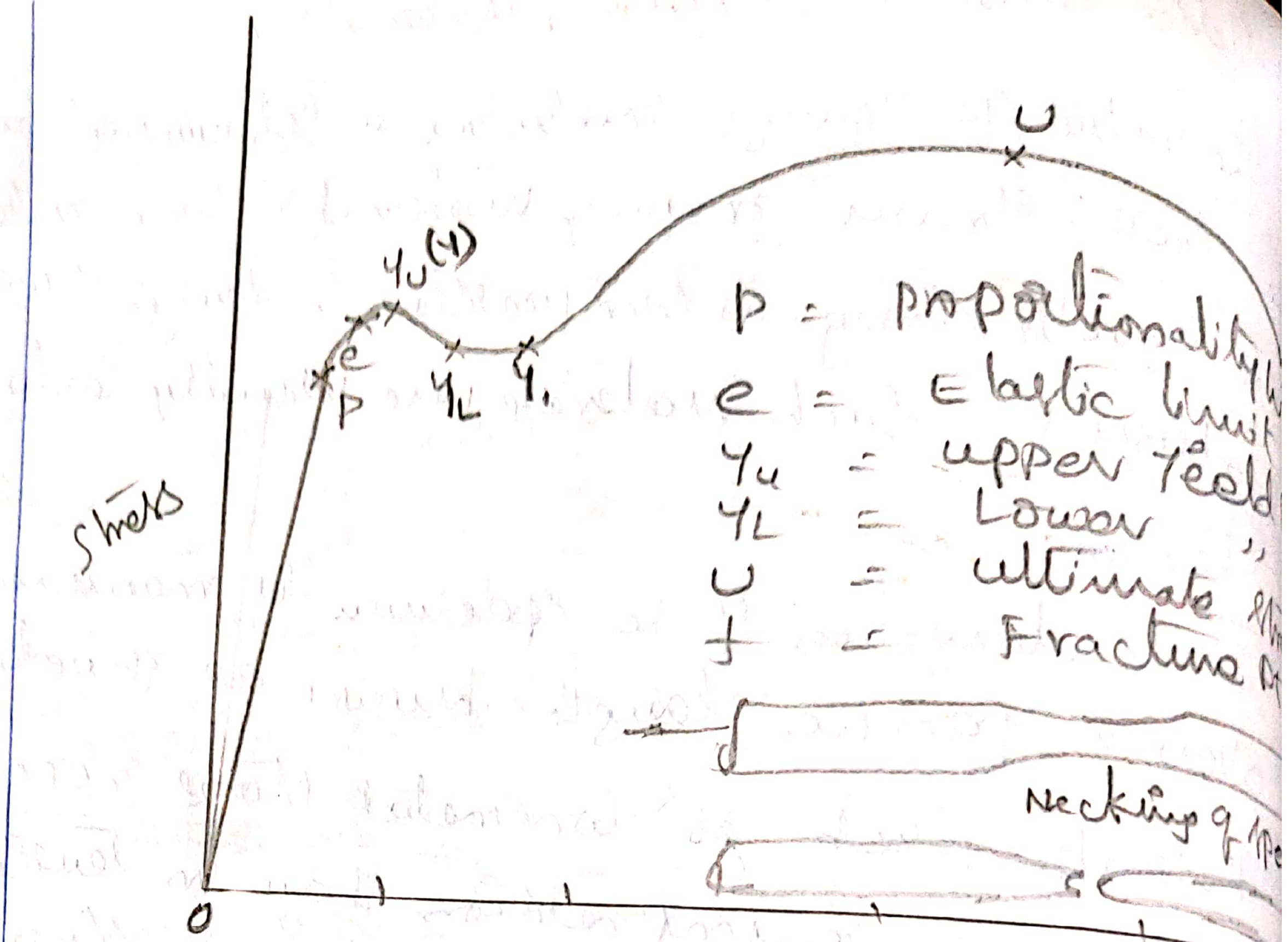
Stress - strain behaviour of materials

The value of young's modulus is determined from the stress - strain graph of material. Some materials are equally strong in compression & tension (metals & alloys). Such materials are usually tested in tension.

The elongation of the specimen is measured over a specific length known as gage length. Materials such as Concrete, Stones & bricks are stronger in compression than in tension are tested in compression. Stress - strain values are plotted in the form of graph, & the value of young's modulus is determined from the slope of curve for any stress value.

Stress - strain diagram for mild steel:

The stress - strain behaviour of mild steel is of specific interest to engineers b/c of the extensive use of mild steel as construction material the behavior is also significant b/c no other material has all the characteristics of mild steel.



Stress-Strain characteristics of mild Ste

A linear relationship b/t stress & strain can be observed initially; the linearity indicates Hooke's law is obeyed upto a certain value.

The Hooke's law is obeyed is known as σ_p, the proportionality limit. The material, however, remains elastic [undergoes elongation vanishes completely on the removal of load] even beyond the limit of proportionality.

σ_e: Elastic limit: The stress upto which the material remains elastic is known as σ_e, the elastic limit.

If the specimen is loaded beyond its elastic limit, it will not regain its original dimension [or plastic] strain on unloading.

If the specimen is loaded further beyond the elastic limit (point e), it reached a point where a sudden increase in strain occurs without an appreciable increase in stress. This feature is known as yielding of the material and is represented by 'y' in (fig). The corresponding stress is known as upper yield point [σ_{yu}]. (σ_{uy}),

A careful testing of the specimen would reveal that the curve drops slightly at this point to yL The corresponding stress is known as lower yield point (σ_{Ly}).

The region y - y, is not steady and displays wariness when tested with precision equipment. Beyond y, further straining of material can occur only with an increase in applied load. This phenomenon is known as strain hardening of the material. The stress-strain continues to rise till u, the max stress the material can sustain. This value is known as ultimate stress (σ_u). of the material.

It would be interesting to note that the b/t points 4 & 4 is about 100 times the b/t 0 & 4. ie The plastic strain of the its ultimate stress is about 100 times its elastic strain.

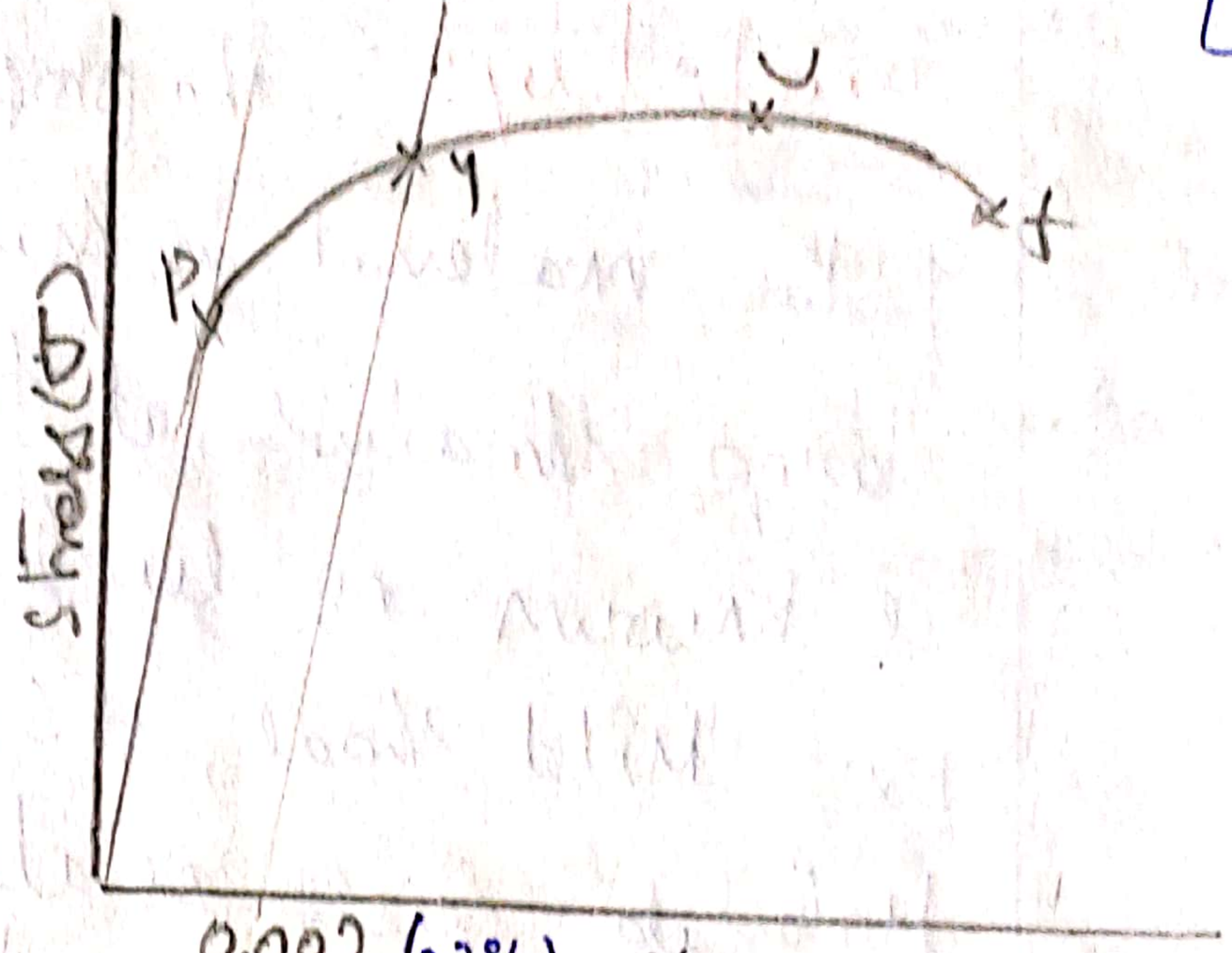
As the specimen is stressed further strain (elongation) increases with a decrease in stress (load) accompanied by contraction (decrease in bar dia). Then is fastens stress reduced considerably at the section of fra known as necking.

Engineers stress - strain: It is general pra to compute stresses & strains with ref to ori [gage length & CS area] of the dimensions. These parameter that dont consider increa & reduction in area are known as engine and engineering strain. [(or) simply stress & str These values are also known as nominal & nominal strain.

When the changes in the dimensions considered in the computations, these are termed true stress & true strain useful in plastic analysis of structure

High Strength Steels

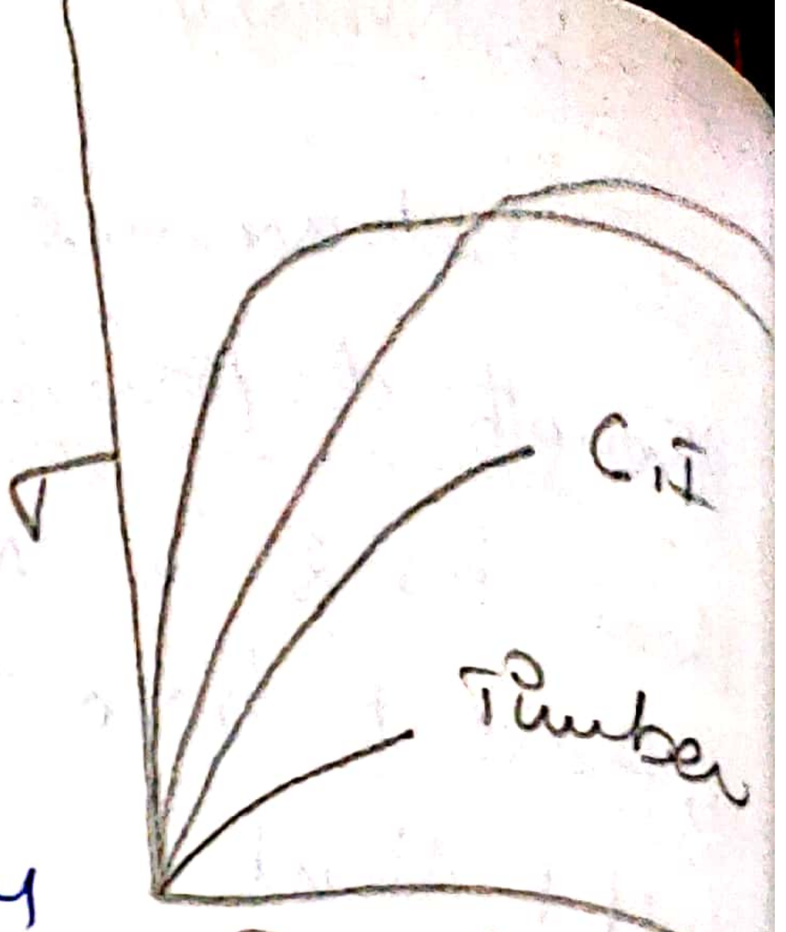
The behaviour of High Strength deformed (HSD) Steel bars is much different from that of mild Steel.



The diagram doesn't indicate a well defined yield point unlike that of mild Steel. The material displays linear characteristics initially (upto P). However the curve flattens out gradually as there is increased beyond 80% of its ultimate value.

The yield Stress (σ_y) for such materials without a well defined yield point is determined from the criterion of Proof Stress.

Proof Stress is defined as the Stress which induces a specified Residual Strain (usually 0.2%) in the material. This is sometimes known as 0.2% proof Stress or simply proof Stress. A line parallel to the initial straight line part of the curve is drawn from 0.2% Strain on the x-axis to intersect the Stress-Strain curve at y, which yields the proof Stress of the material.



Ductility: The property of the material to sustain large strains at fracture is known as ductility

Ex: Mild steel, H.S.D.

Materials such as aluminium at mild steel, with elongation at failure greater than 15% are termed d

The ductility of a material is usually exp interms of % elongation at fracture.

$$\%\text{age elongation} = \frac{L_f - L_o}{L_o}$$

where L_f = Length @ fracture

L_o = original length.

Ductility is sometimes expressed as % of C/S area at fracture

$$\%\text{age Contraction} = \frac{A_o - A_f}{A_o}$$

where A_o = original area of C/S

A_f = area of C/S @ fracture.

Elastic & plastic ranges: The region of the stress curve extending from the origin to the proportionality limit is known as elastic of material. The region of curve extending the proportionality limit to the point of fra is known as plastic range of material

Ex: Ductile materials have a large plastic Brittle materials can't sustain much stress

Factor of Safety: The design of a structure is based on several assumptions regarding loads & materials. It is necessary to limit the actual stress to a value much smaller than the yield stress of the material because of various uncertainties of design parameters. Some of uncertainties

Loads: on structure cannot be estimated with any degree of exactness. The dead loads (self wts) of a structure may be different from assumed value b/c of possible deviations from ,, dimensions of members & unit wts of materials. Live loads are even less certain.

Material characteristics: may be different from values assumed in the design. Materials are assumed to be isotropic and homogeneous, and to possess the same properties in tension & compression. Such assumptions may not be valid in the case of materials such as concrete & cast steel metals.

Hookes law: is assumed in the analysis. However strains are not in direct proportion to stresses in some materials such as concrete & non-ferrous alloys.

In order to account for the uncertainties mentioned above the yield stress of a material is reduced by a factor while specifying the allowable stress (more permissible); this factor is known as the

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The working stress (or) safe stress (or) allowable stress (or) design stress) of a material is computed by dividing the yield stress by the specified factor of safety. The actual stress in a material should not exceed its working stress. (or) under working condition.

$$F.O.S = \frac{\sigma_y}{\sigma_w} \text{ (or) } \sigma_w = \frac{\sigma_y}{F.S}$$

NOTE: A high factor of safety value is adopted for important buildings such as monuments, public buildings, hospitals & structures needed during of disasters.

Temporary structures (Cradle forms, temporary sheds, ware houses & cattle sheds may be designed with a lower factor of safety.

Elongation of a bar: $\epsilon = e = \frac{\sigma}{E} = \frac{P}{AE}$

∴ Elongation of a bar of length l. Δl (or) $\delta l =$

∴ Elongation of bar is proportional to the load on its length, inversely proportional to area of cross

Principle of Superposition: If the proportionality between the resultant stresses in a material & the P.O.S.P is used.

It states that the effect of a set of loads on is the same as the sum of the effects caused by each load acting separately.

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STRENGTH OF MATERIALS

Chapter 1

Mechanical properties: The following are the most important mechanical properties of Eng. materials:

1) Elasticity 2. Plasticity 3. Ductility
4) Brittleness 5. Malleability 6. Toughness
7) Hardness 8. Strength.

Elasticity: when external forces are applied on a body, made of engineering materials the external forces tend to deform the body while the molecular forces acting between the molecules offer resistance against deformation. The deformation (or) displacement of the particles continues till full resistance to the external forces set up. If the forces are now gradually diminished, the body will return, wholly (or) partly to its original shape.

Def: Elasticity is the property by virtue of which a material deformed under the load is enabled to return to its original dimensions when the load is removed.

If the body regains completely its shape, it is said to be perfectly el

For any particular material, a crite value of the load, known as the elastic li marks the partial breakdown & elasticity which removal of load results in a de permanent deformation (or permanent

Ex: Steel, aluminium, copper, clone etc may be considered to be perfectly within certain limits.

STRESS - STRAIN RELATIONSHI

The load per unit area, normal to the load is known as stress (ϕ). Similarly deformation per unit length in the dire of " is known as strain (ϵ).

The elastic properties of materi in engineering are determined by te equipped with testing m/c's capable the specimens in gradually applied and the resulting stresses & strains a

at all such load increments, till to specimen fails.

Fig 1.

In Fig 1. the specimen is loaded only upto point A which will within the elastic limit E. when the load, corresponding to point A, is gradually removed the curve follows the same path AO. and the strain completely disappears. Such a behaviour is known as the elastic behaviour.

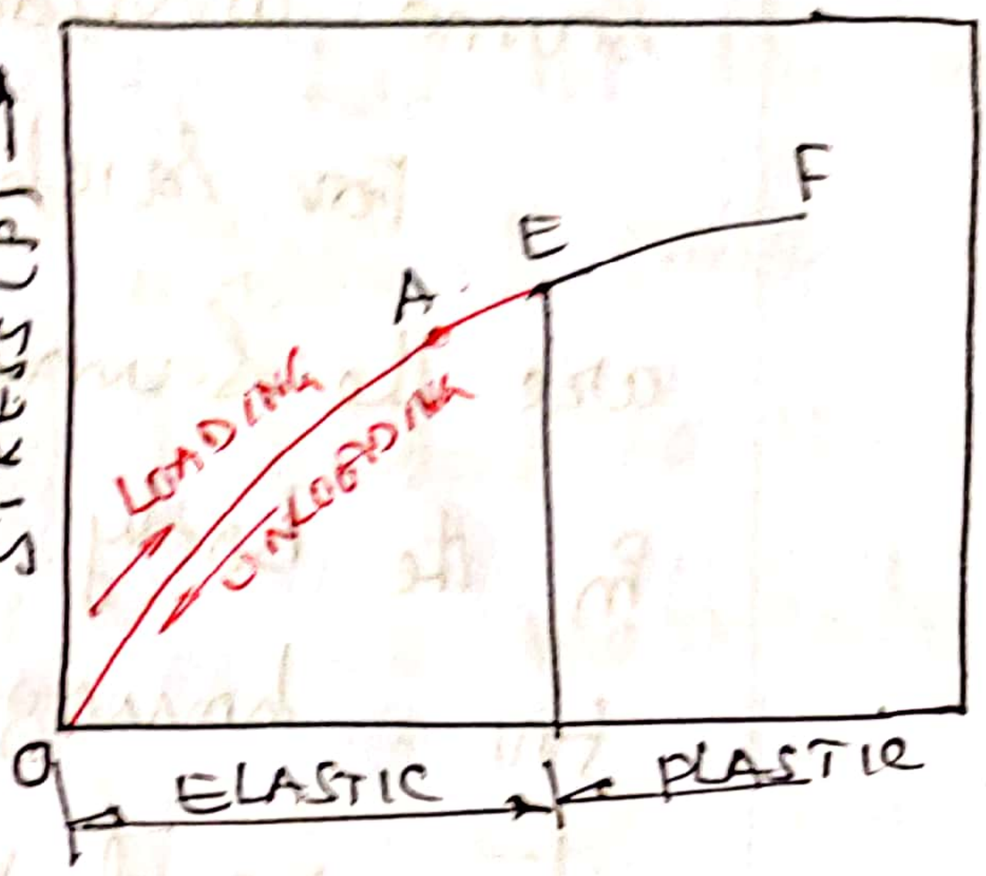
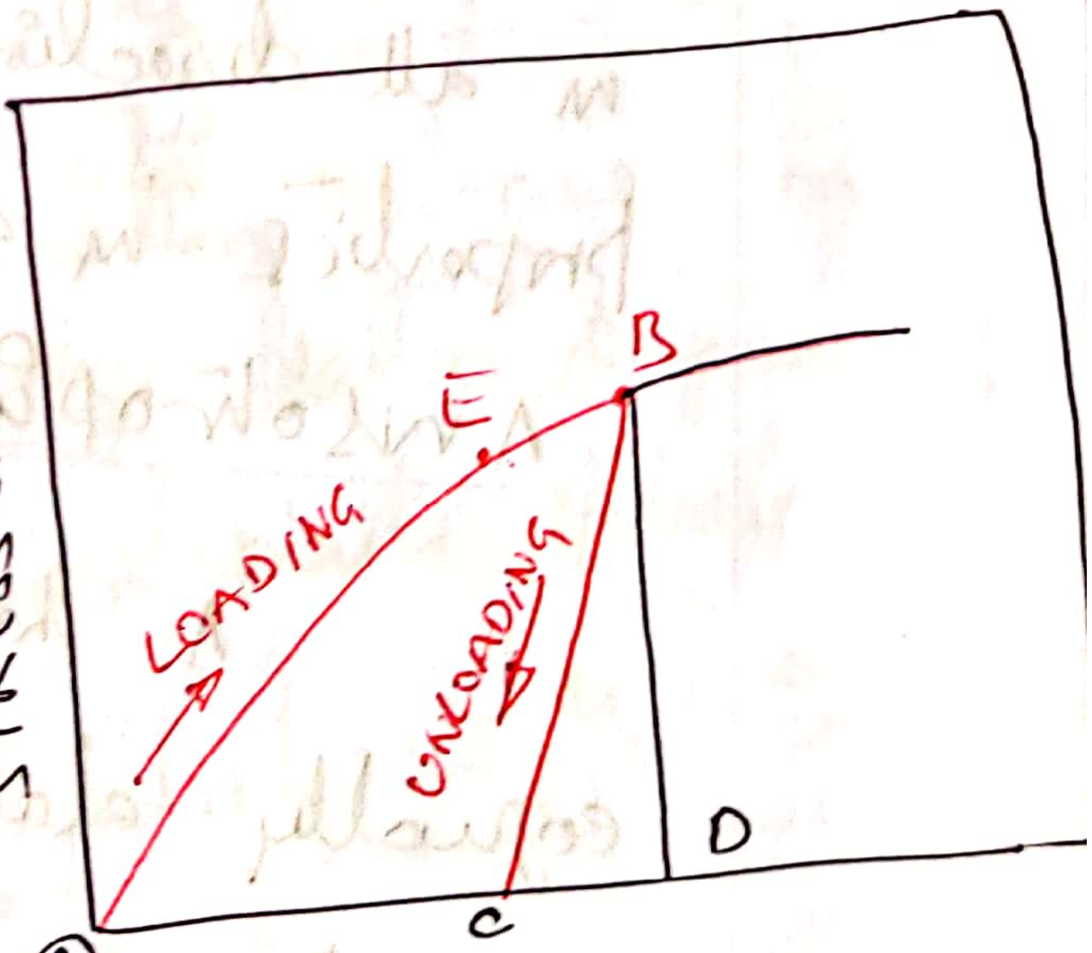


Fig 2:

In Fig 2, the specimen is loaded upto point B, beyond the elastic limit E, when the specimen is gradually unloaded, the curve follows path BC, resulting in a residual strain (OC) (or) permanent strain. Such a behaviour of the material, loaded beyond the elastic limit, is known as partially elastic behaviour.



HOMOGENEITY AND ISOTROPY

A material is homogeneous if it is of same composition throughout the body. For such a material, the elastic properties are the same at each and every point in the body. It is interesting to note that for a homogeneous material, the elastic properties need not be the same in all the directions.

If a material is equally elastic in all the directions, it is said to be Isotropic.

If however, it is not equally elastic in all directions i.e it possesses different properties in different directions, it is called Anisotropic.

A theoretically ideal material can be equally elastic in all directions i.e isotropic. Structural materials meet the requirements of homogeneity and isotropy.

PLASTICITY

Plasticity is the converse of elasticity. A material in plastic state is permanently deformed by the application of load, and it has no tendency to recover. Every elastic material possesses the property of the plasticity. under the action of large forces, most engineering materials become plastic and behave in a manner similar to a viscous liquid.

The characteristic of the material by which it undergoes inelastic strains beyond those at the elastic limit is known as plasticity.

when large deformations occur in a ductile material loaded in the plastic region, the material is said to undergo plastic flow.

The property is particularly useful in the operations of pressing or forging

plasticity is also useful in the design of structural members, utilising its ultimate strength.

MECHANICS (OR STRENGTH) OF MA

Three fundamental areas of engineering mechanics (or applied mechanics) are

(i) Statics (ii) Dynamics

(iii) Mechanics (or strength) of materials

Statics and dynamics are devoted mainly to the study of the external effects of forces on rigid bodies i.e the bodies for which the change in shape (or deforming) may be neglected.

In Contrast to this, mechanics of materials commonly known as strength of materials deals with the relation between externally applied forces and their internal effects on the bodies.

The solid bodies include axially loaded members, that is rods, thin as well as thick cylinders and shells, beams and columns, and shafts. Structures that are assemblies of these components. These bodies are no longer considered to be rigid, the deformation, however

are of major interest.

A m/c part (or) structure should neither break nor deform excessively.

The purpose of studying strength of materials is to ensure that the structure used will be safe against maximum internal effects that may be produced by any combination of loading.

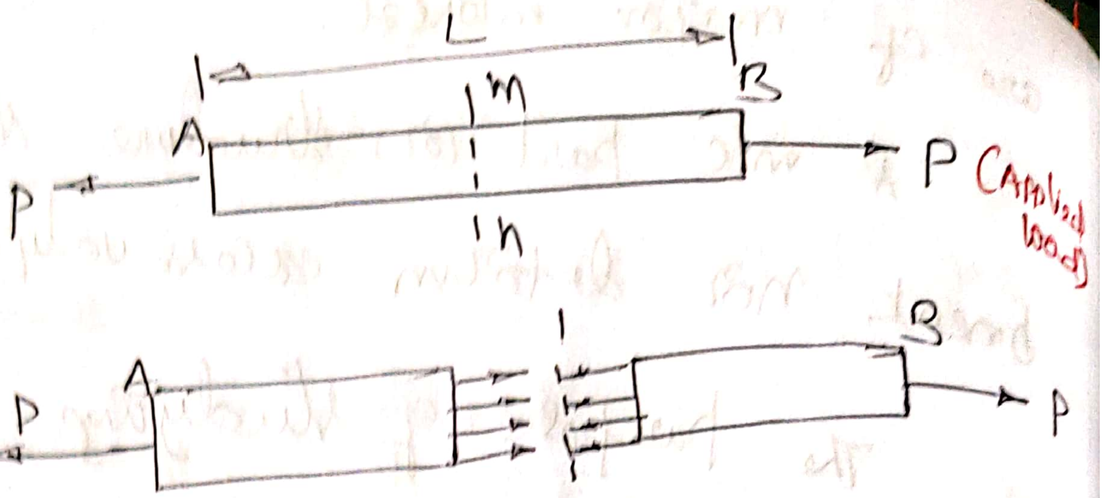
SIMPLE STRESSES AND STRAINS

When a body (ie structural element) is acted upon by external force (or load, internal resisting force is set up. Such a body is then said to be in a state of stress.

Stress is the resistance offered by the body to deformation.

For further understanding of this internal resistance, Consider a prismatic bar AB subjected to axial forces at the ends.

fig 1



A prismatic bar is a straight member of uniform C/S (A) throughout length (L).

In order to know the internal stresses in the prismatic bar, take a section normal to the longitudinal axis of the bar. Such a section is known as C/S.

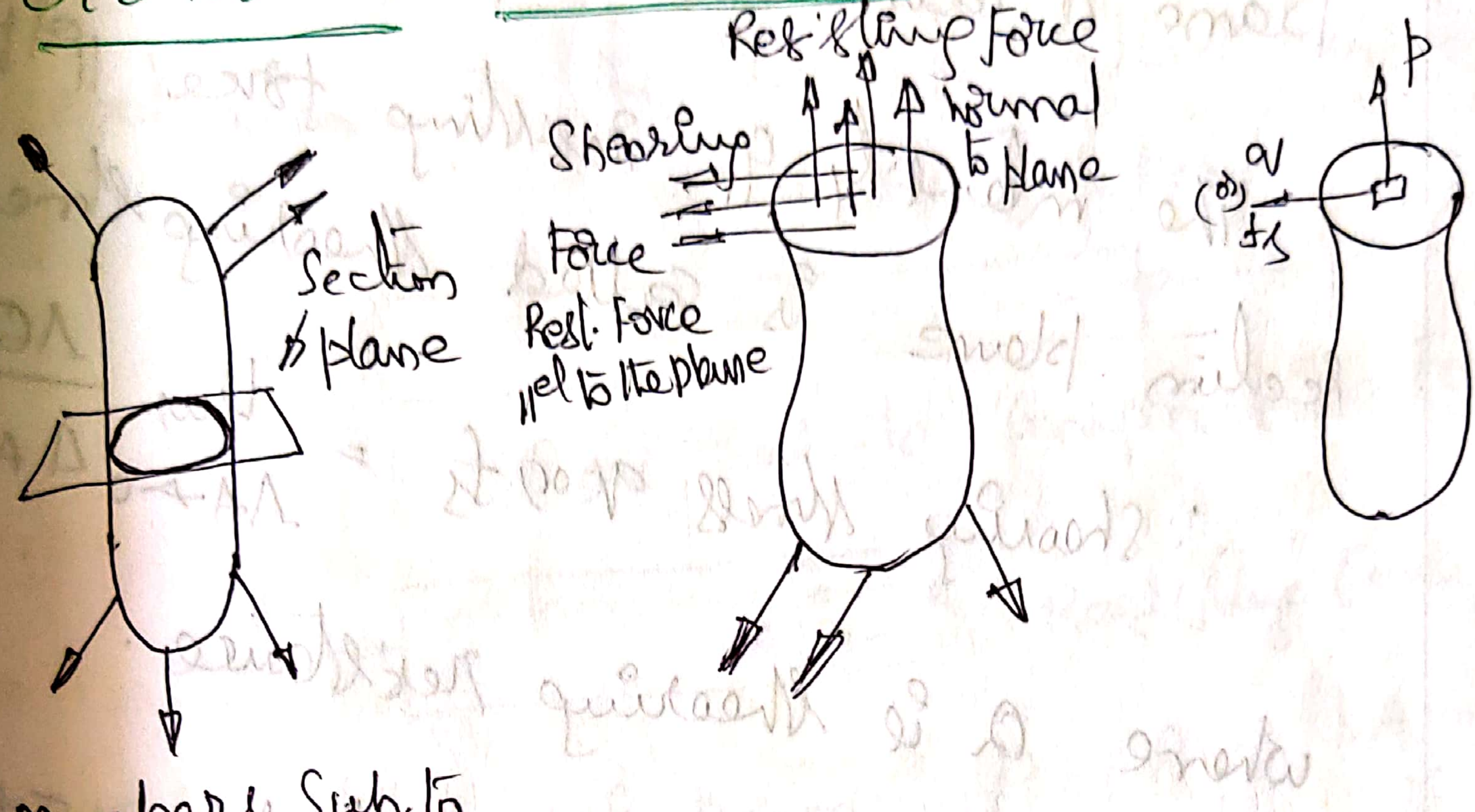
If we consider the equilibrium of left part or right part at section mn as a free body, the internal resistance the stress (P) offered by the molecules to the external force may be assumed uniformly distributed over the whole C/S. Then $\sigma = \frac{P}{A}$

Where, P = Internal resistance = Stress

A = Area of C/S normal to axis.

As the stress (V) @ P acts in a direction ⊥r to the cut surface, it is referred to as a normal stress.

GENERAL MEANING OF STRESS



@ Members Sub. to Forces

Normal Stress σ@P = $\lim_{\Delta A \to 0} \frac{\Delta R}{\Delta A} = \frac{dR}{dA}$ —(1)

where R is normal resisting force

When a member is subjected to loads it develops resisting forces. To find resisting forces developed a section plane may be passed thro the member and equilibrium of any one part may be considered. Each part is in equilibrium under the action of applied forces and internal resisting forces. The resisting forces may be conveniently split into normal & parallel to the section plane.

The resisting force acted to the plane is shearing resistance

The resisting force normal (Rs) to the plane is called intensity of normal stress

The intensity of resisting force acted to section plane is called shearing stress.

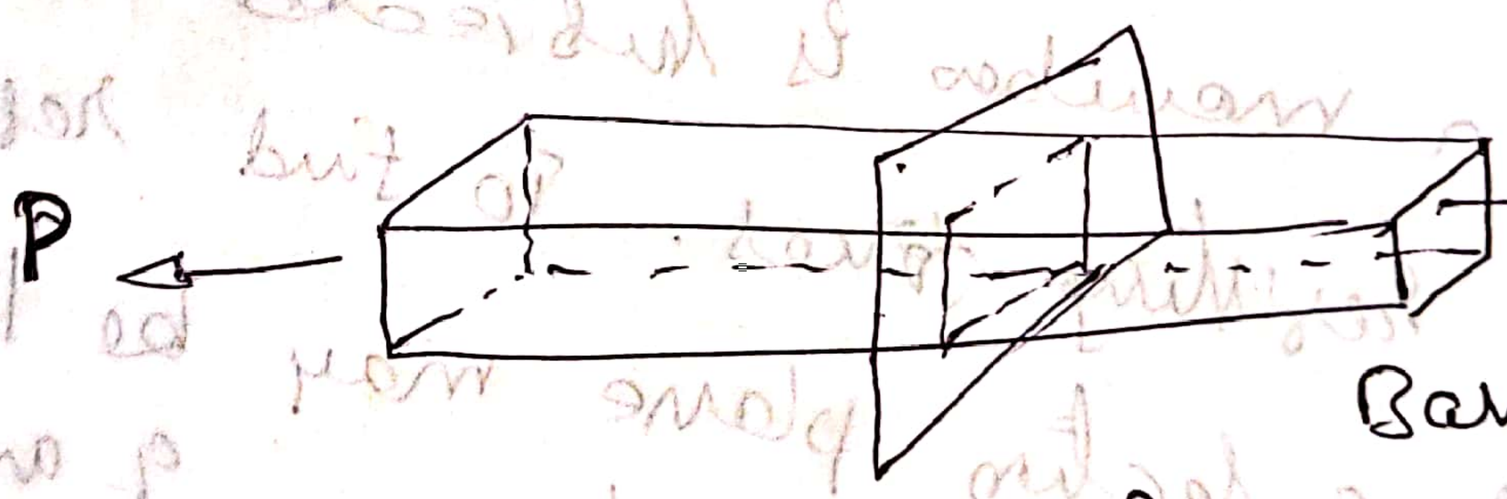
$\therefore$ Shearing stress $\tau = \lim_{\Delta A \to 0} \frac{\Delta Q}{\Delta A}$

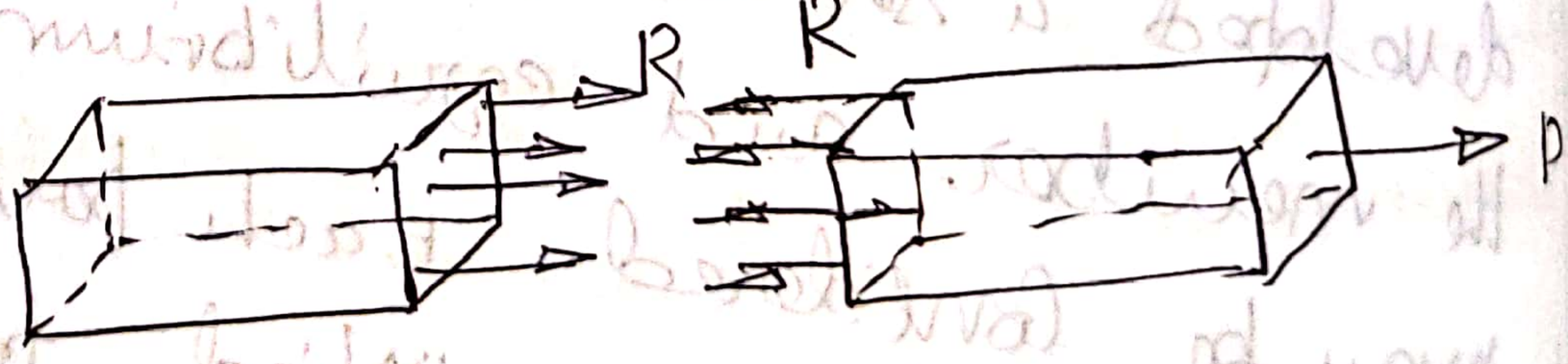
where Q is shearing resistance.

From eqn 1 & 2, $dR = p\,dA$ (or) $\tau\,dA$

$R = \int p\,dA$ (or) $\int \tau\,dA$

$Q = \int q\,dA$





Resisting Force developed

Considering equilibrium of a cut piece of the

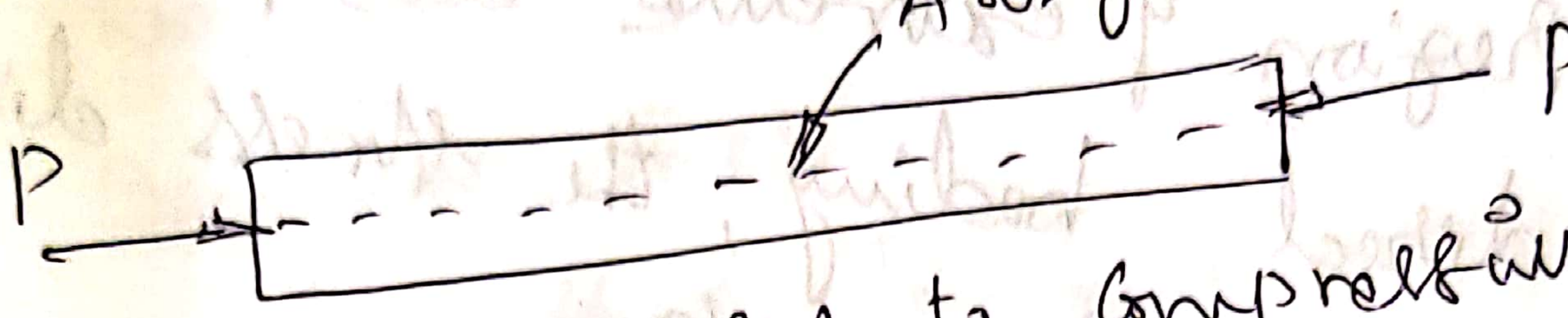
$P = R = \int \sigma\,dA = \sigma \int dA = \sigma$

From (3) $P = \int p\,dA = p \int dA$

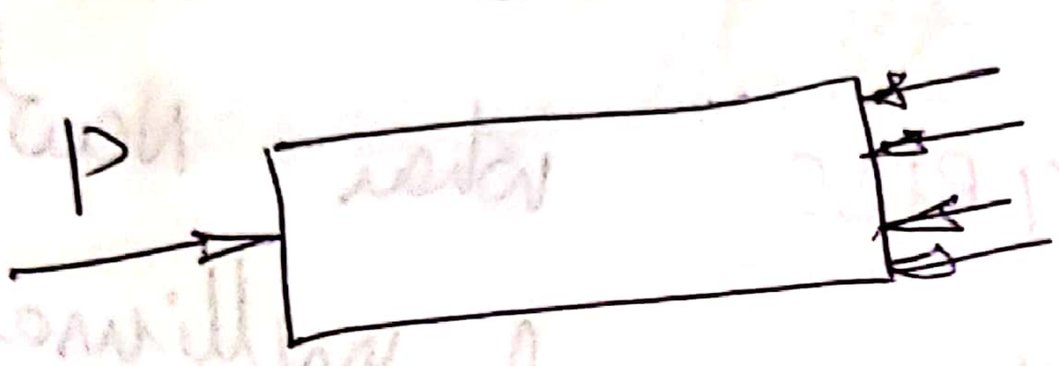
$= \sigma \int dA$

$\therefore \boxed{p = \frac{P}{A}}$ (or) $\sigma =$

∴ In case of axial load P, the stress developed is equal to the load per unit area. under this type of normal stresses the bar is being extended. Such stress which is causing extension of the bar is called Tensile stress



Bar Sub. to Compressive Forces

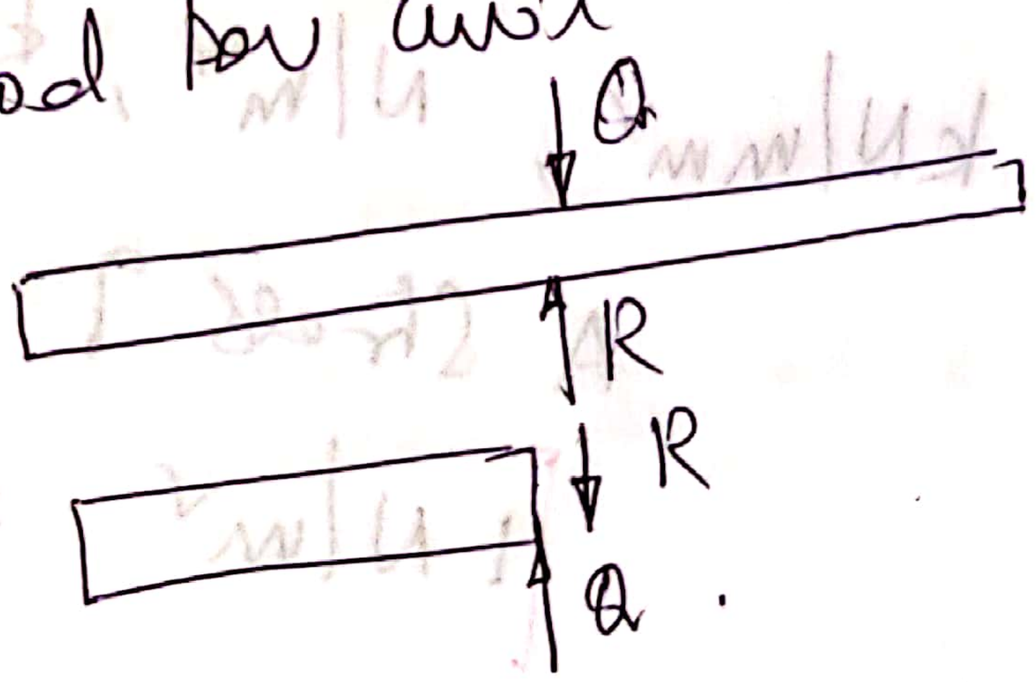


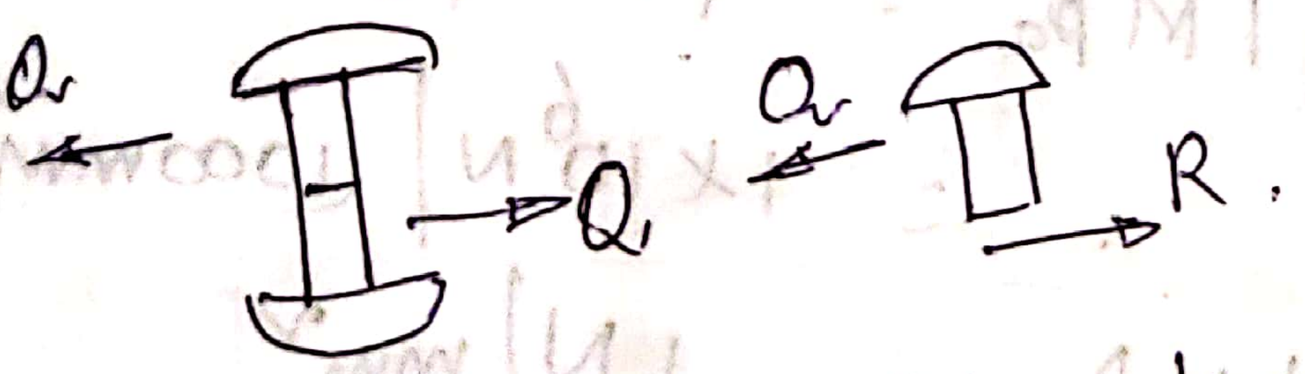
Resisting Force developed

For equilibrium of piece $P = R = \sigma A$

∴ whether it is tensile (or) Compressive, the stress developed in a bar Sub. to axial forces is equal to load per unit area.

Shear Stress:





Rivet in Direct Shear

Shearing force (or) the Shear Stress

Let Q be the Shearing force

under equilibrium $Q = R = q_r A$.

$$q_r = \frac{Q}{A}$$

∴ Direct Shear Stress is equal to Shearing force / unit area

SAINT VENANT'S PRINCIPLE

We have assumed above that the distribution of stress over the c/s mn is uniform. This is true only when Saint-Venant's principle is based on

This principle states that except in the region of extreme ends of a bar carrying direct loading, the stress distribution at the c/s is uniform

UNITS OF STRESS:

As unit of force and millimetre as unit of length, unit of stress will be N/mm^2. The other derived units used in practice are kN/mm^2, N/m^2, kN/m^2 (or) M N/m^2.

A Stress of 1 N/m^2 is known as Pascal

$$1\ N/m^2 = \text{Pascal} = Pa$$

Hence $1\ MPa = 1\ MN/m^2$

$$= 1 \times 10^6\ N / (1000\ mm)^2$$

$$1\ MPa = 1\ N/mm^2$$

KINDS (or) TYPES OF STRESSES

There are the following kinds of stresses

1) Normal stresses
   (a) Tensile stress
   (b) Compressive stress
2) Shear stress (or) Tangential stress
3) Bending stress
4) Twisting th (or) Torsional stress
5) Bearing stress.

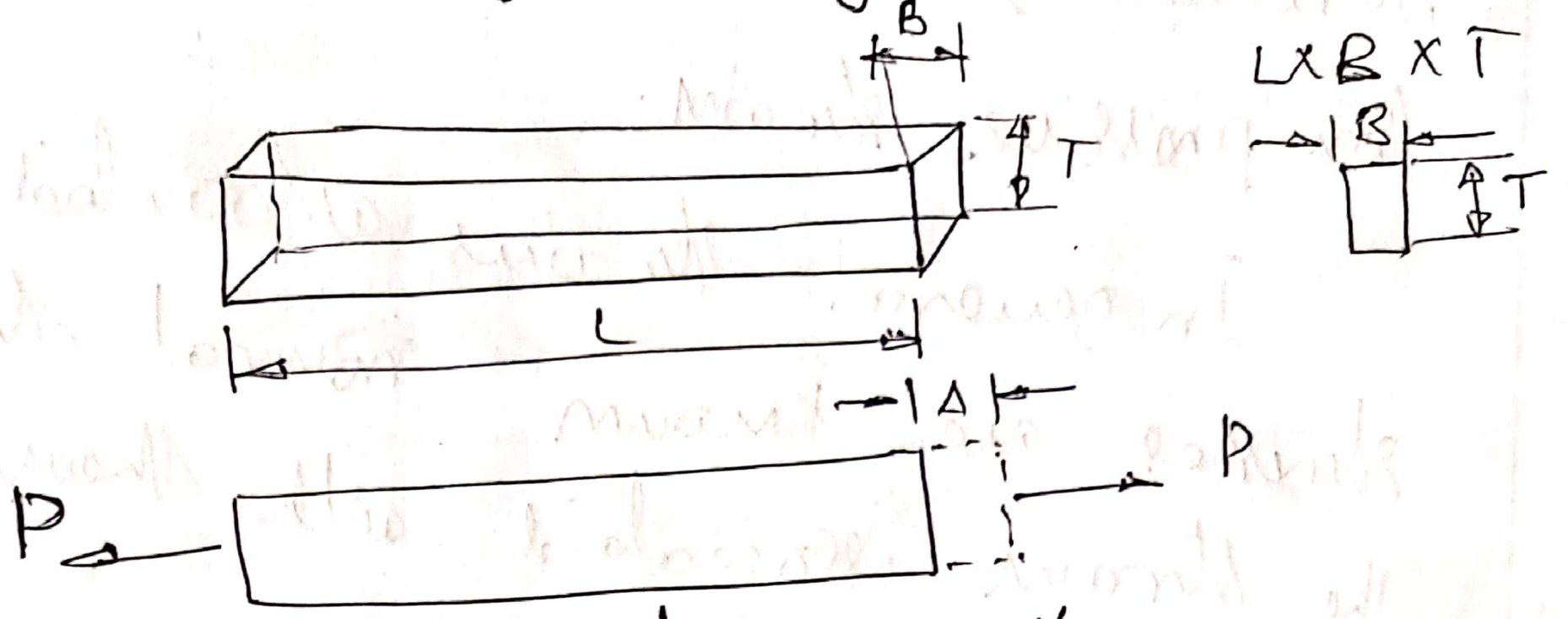
Shear stress: Shear stress is the one which acts parallel (or) tangential to the surface. Thus shear stress exists b/t the parts of a body when the two parts exert equal and opposite forces on each other laterally in a direction tangential to their surface in contact.

Lateral: Connected with side of something (or) with a movement to the side.

Exi- Direct shear stresses arise in bolted, pinned, riveted, welded (or) glued joints.

STRAIN

When a prismatic bar is subjected to axial load, it undergoes a change in length, as indicated in fig



Deformation and Strain

This change in length is usually called deformation If the axial force is tensile, the length of the bar is increased, while if the axial force is compressive, there is shortening of the length of the bar.

This elongation (or shortening) as the case may be, is the cumulative result of the stretching (or compressing) of the material throughout the length of the bar.

The deformation i.e elongation (or shortening) per unit length of the bar is termed as Strain (ε or e)

STRAIN: In general strain is the measure of the deformation caused due to external loading.

If the bar is in tension, the resulting strain is known as tensile strain. Similarly the resulting from a compressive force is known as compressive strain.

In general strains associated with stresses are known as normal strains. Similarly the strain associated with shear stress is known as shear strain.

Since strain is the deformation per unit it is a dimensionless quantity. It is expressed as pure number.

Ex: If the deformation of a bar of 1.6 m length is 1.2 mm, the strain $\varepsilon = \frac{\Delta}{L} = \frac{1.2\ mm}{1.6 \times 1000\ mm} = 0.00075$

Strain is sometimes recorded as mm/m or μ

∴ In above example strain is 0.75 mm/m or 750

Ex: A prismatic bar has a cross-section of 25 mm and a length of 2 m. Under an axial tensile force of 90 kN, the measured elongation of the is 1.5 mm. Compute the tensile stress and

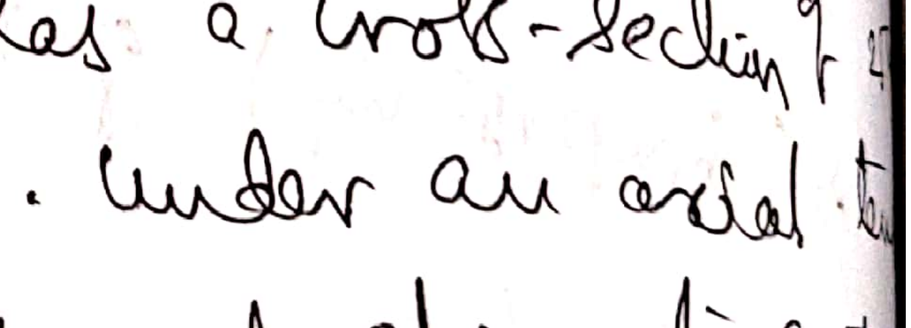
Sol:

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

$$= \frac{90 \times 1000}{25 \times 50}$$

$$= 720\ N/mm^2$$

$$= 72\ MPa$$



$$\left[1\ N/mm^2 = 10^6\ N/m^2 \right.$$

$$\text{Strain } \varepsilon = \frac{\Delta}{L} = \frac{1.5}{2 \times 1000} = 0.75 \times 10^{-3} =$$

STRESS - STRAIN DIAGRAM

A = proportional limit
B = Elastic limit
C = upper yield point
C' = Lower "
E = ulti-mate strength
F = Rupture strength (Break)

oa = Linear deformation
ob = Elastic "
bd = perfect plastic zone
de = Strain hardening
ef = Necking

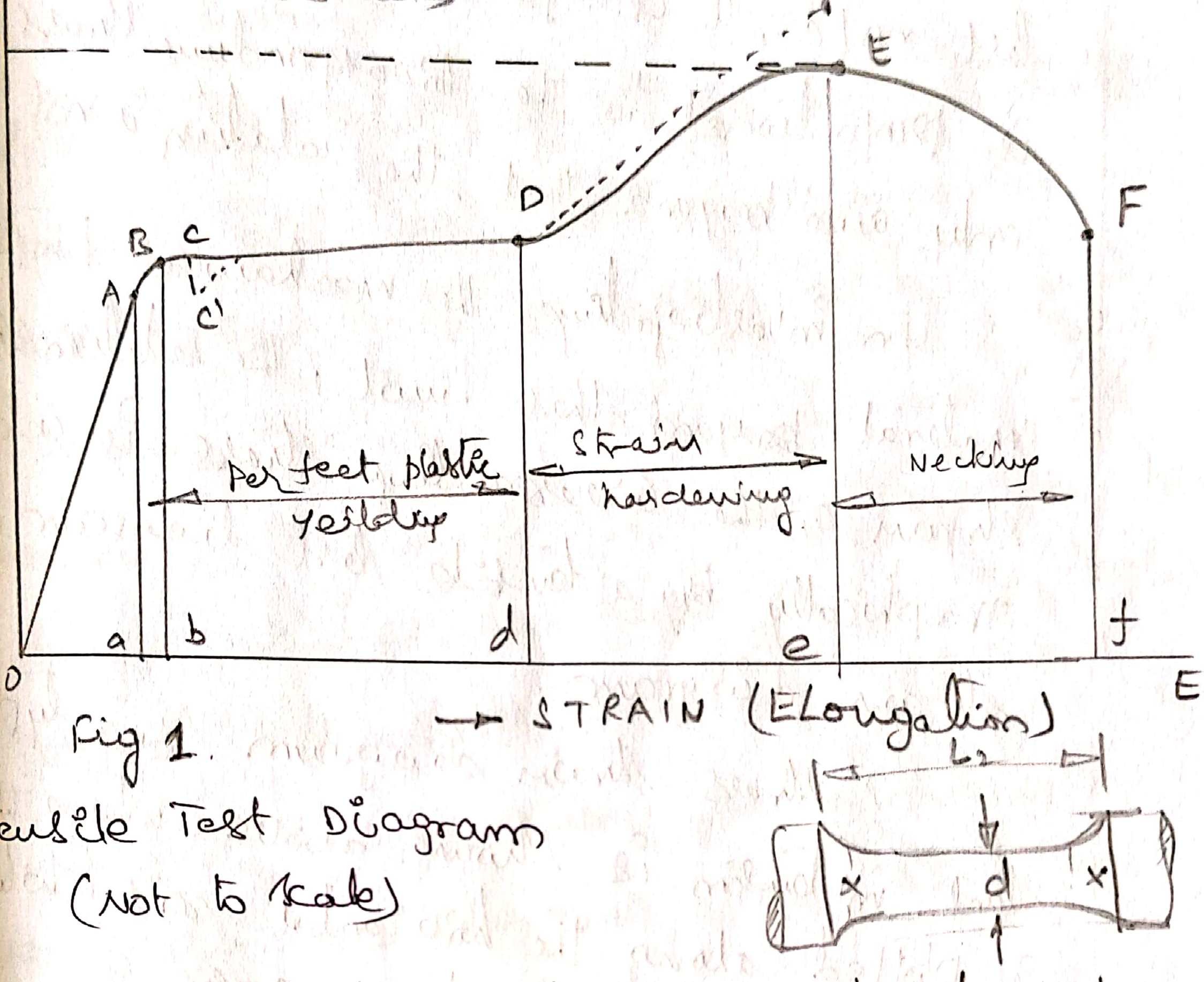
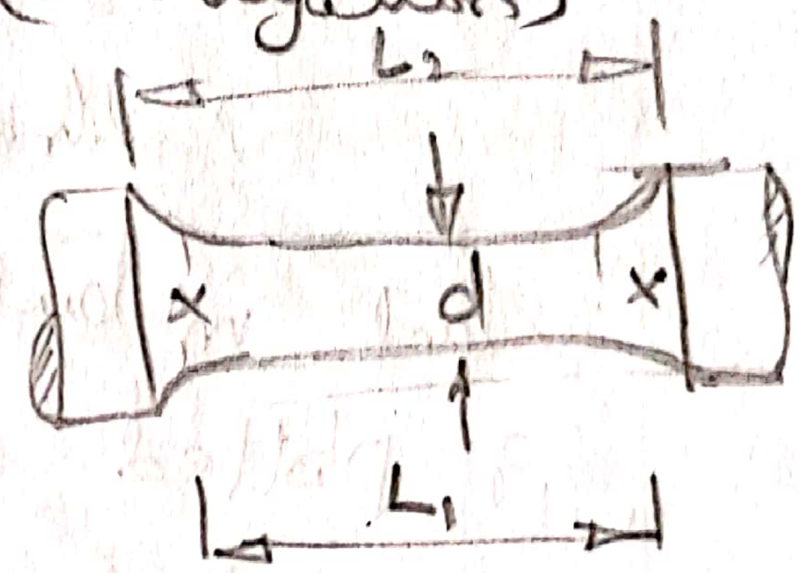


Fig 1.

Tensile Test Diagram (Not to scale)



L_1 = Length (Gauge length) of specimen
d = dia of specimen

The Mechanical properties of a material, are determined in the laboratory by performing tests on small specimens of the material, in the material testing laboratory.

The most common materials test is the tension test performed on a cylindrical specimen [of gauge length 'L' & dia 'd'] of the material.

* Gauge Length (L_1): The length over which the extension is measured

L_2: Grip to Grip length of specimen

The loads are measured on the machine dial while the elongations are measured with the help of Extensometers.

When such a specimen of a ductile material is subjected to a gradually increasing pull in a test m/c, it is found that the resultant stress is proportional to the corresponding strain upto a limit only and beyond that, the relation is not linear.

In investigating the mechanical properties of material beyond this limit, the relationship between stress & strain of the corresponding stress is usually shown graphically by a tensile test diagram (or) stress strain diagram.

A stress-strain diagram for a typical Mild steel in tension is shown above, where the stress is plotted along the horizontal axis while the strain is plotted on the vertical axis.

The diagram begins with a straight line OA, in which the stress & strain relation is linear, i.e stress and strain are directly proportional. The point A marks the <u>limit of proportionality</u> beyond the curve becomes slightly curved, until point B, the <u>elastic limit</u> of material, is reached.

Region AB is the non-linear region in which the stress is not proportional to strain, and the elongation increases more & rapidly.

upto the point B, the removal of load would result in complete recovery by the specimen of its original dimensions.

If the load is increased further, yielding takes place, point C is the point of sudden large extension, known as yield point.

After yield point stress is reached, the ductile extensions take place, the strains increasing at an accelerating rate as represented by C to D.

During the ductile extension, the area of CDS practically the same proportion that decreases in the length increases, and hence this is the region of perfect plasticity or yielding. (CDJ).

perfect Plastic means, the material can deform without an increase in the applied load. For mild steel, the elongation in this region is 10 to 15 times the elongation that occurs b/t OA.

If the load is further increased, the steel begins to strain harden. During strain hardening the material appears to regain some of its strength and offers more resistance, requiring increased load for further deformation. This is because the material undergoes changes in its atomic & crystalline structure in this region.

After D, with further increases in loads ... the point E of the maximum load (or) ultimate ... [commonly known as the ultimate strength] is ... upto maximum load, the bar extends uniformly over its parallel length but of straining ... a local reduction in c/s area occurs (ie formation of waist) and as the load is concentrated at reduced area, a considerable load extension [known as concentrated plastic deformation] also takes place, till the failure (or) rupture takes place at F.

It is customary to base all the stresses on the original c/s area of the specimen, and as the latter is not constant, the stresses so calculated are known as nominal stresses.

The nominal stress is less at rupture than at max load as indicated by point. The diagram of real stresses [load divided by reduced area of c/s] would be as shown by dotted curve. The breaking load divided by the reduced area of section (ie actual stress) is greater than at the max. load.

STRESS - STRAIN [Tensile test diagram]

For mild steel :

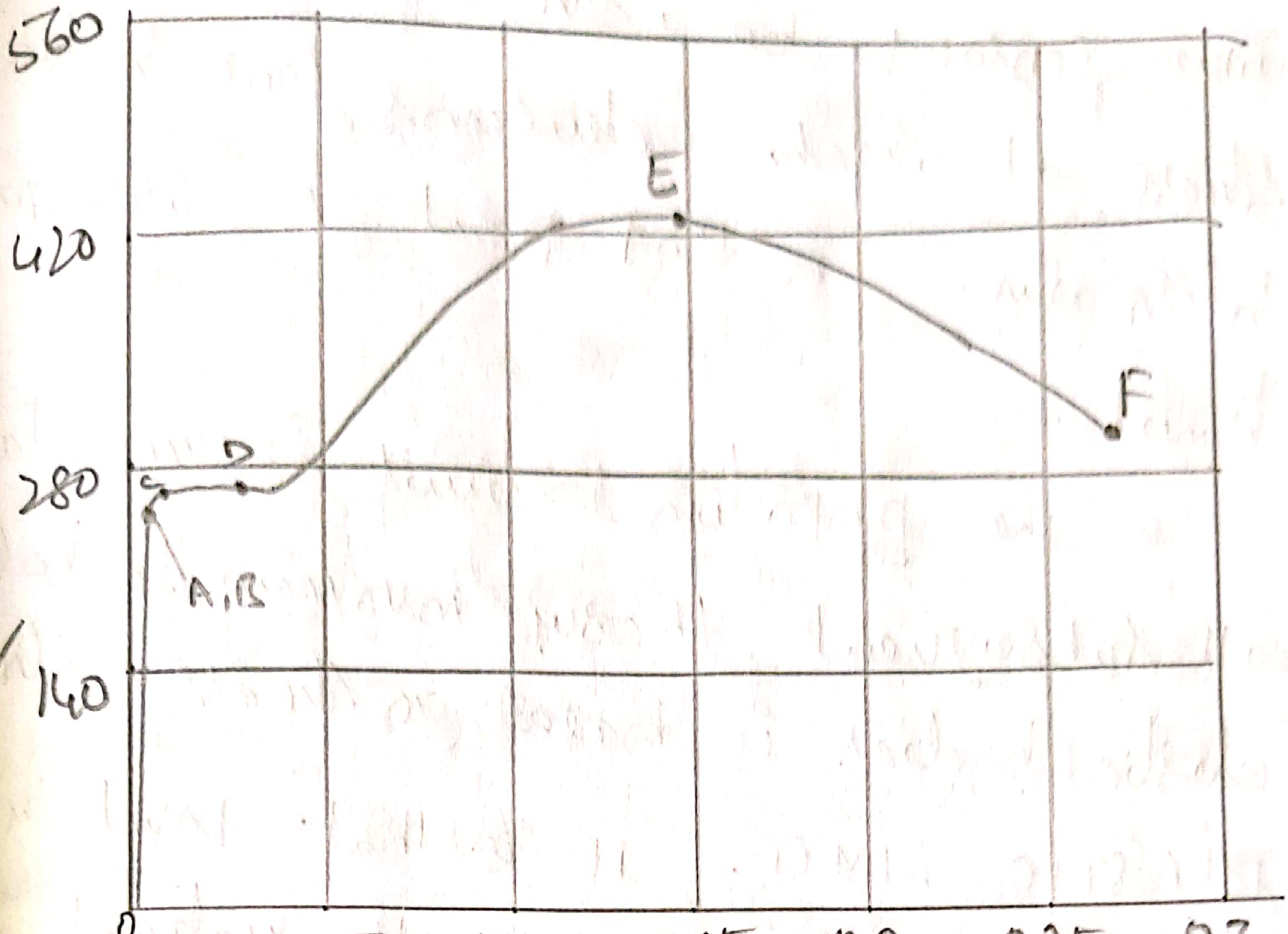


Fig 2.

Fig shows a stress - strain curve in tension for mild steel drawn to scale. The strains that occur from C to D are 15 times more than the strains that occur from O to A, and further the strains from D to F are many times greater than those from C to D. Hence, in this diagram the linear part of the diagram appears to be vertical, with the points A, B and C overlapping

LIMIT OF PROPORTIONALITY.

It is the stress at which the stress-strain curve ceases to be a straight line. It is the stress at which extensions cease to be proportional to strain. In fig 1 point A corresponds to limit.

The proportional limit is important because all subsequent theory involving the behaviour of elastic bodies is based on stress-strain relations.

ELASTIC LIMIT:

It is that point in the stress-strain curve upto which the material regains its original shape i.e the material regains its original dimensions after removal of the load.

Thus elastic limit represents the maximum stress that may be developed during a test such that there is no permanent (or) deformation after removal of the load.

For many materials, elastic limit, proportional limit are almost numerically the same and the terms are sometimes used by many.

In fig 1 ob is the elastic range.

plastic range is the region of stress-strain between the elastic limit (B) and point of rupture (f).

bf is the plastic range in which plastic deformation takes place.

UNIT-2 SHEARING FORCES AND BENDING MOMENTS

INTRODUCTION: Stressed set up in a bar due to axial forces of tension or compression. A tie subjected to axial tension elongates in its length. A strut s.t axial compr. contracts in its length

When a structural member carries lateral forces, two important types of loading action are set up at any section. These are

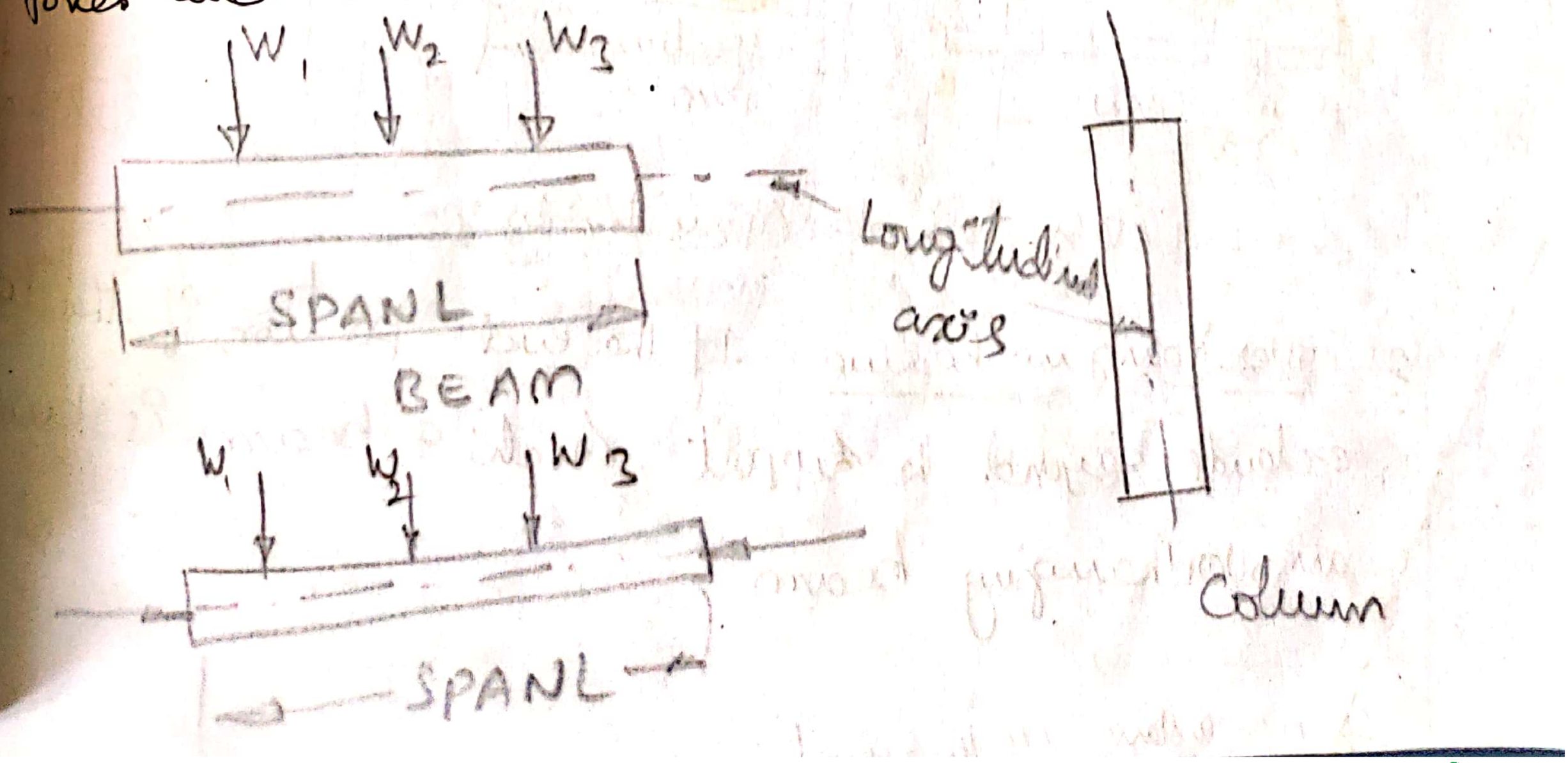
(A) A Bending moment
(B) A shearing force.

The effect of transverse loads ~~is~~ is to bend the structural member over its length.

BEAM: A beam may be defined as a structural member subjected to external loads at right angles to its longitudinal axis.

Column: It the structural members sub. to external forces which act along its longitudinal axis.

BEAM-Column: If a structural member is sub. to both transverse forces and axial forces simultaneously it is called a beam-Co



Note: A beam differs from a bar in that it is subjected primarily to the action of loads acting on them.

Bending: Bending implies the deformation of a bar by load ⊥r to its axis as well as force couples acting a plane passing through the axis of the bar.

TYPES OF BEAMS

Beams are classified depending on the types of supports on which it rests as follows:

1. simply supported beam
2. overhanging beam
3. cantilever beam

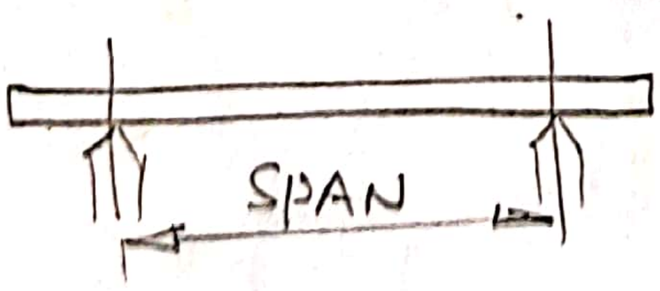
} Statically Determinate beams
(Equilibrium eqns are sufficient to solve the problem)

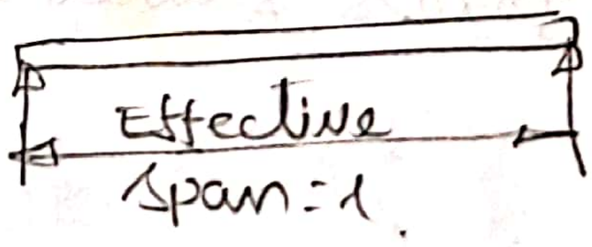
4. Fixed beam
5. continuous beam
6. propped cantilever beam

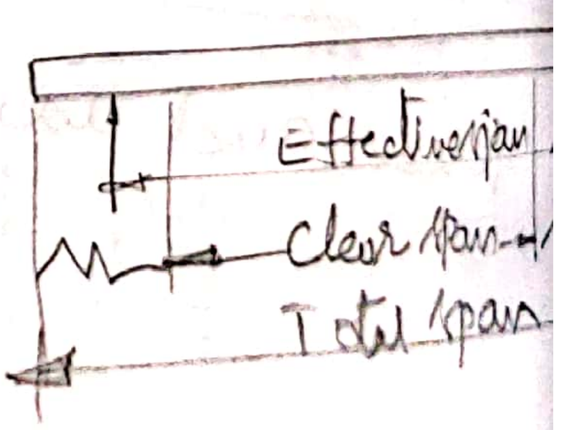
} Statically Indeterminate beams
A structure is statically indeterminate (or hyperstatic) when the static equilibrium eqn's are insufficient for determining the internal forces & reactions

1. Simply Supported Beam: when the two ends of a beam are simply supported, it is called S.S. beam.

A beam with one end hinged and the other simply supported on rollers may also be called simply supported

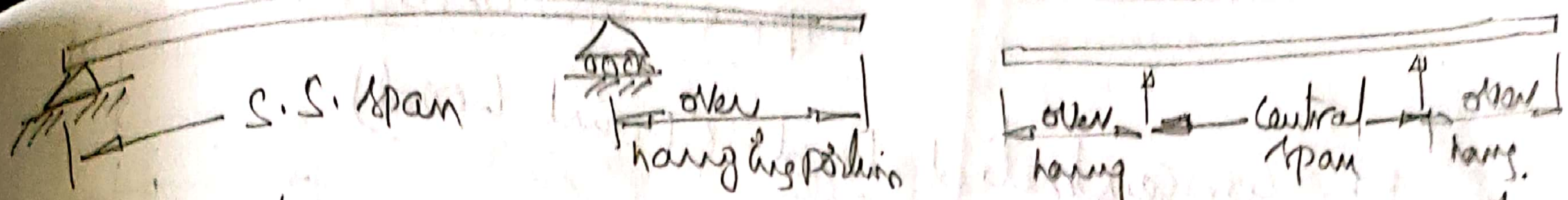






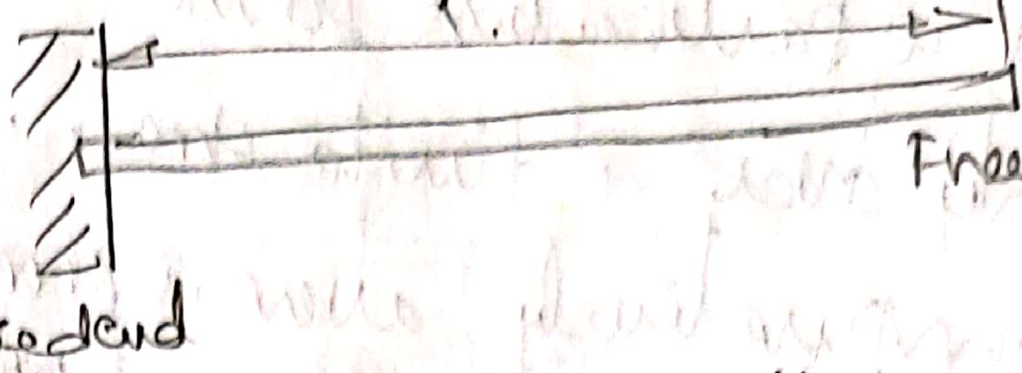
Ex: Roof slabs, simple span Bridges

2. overhanging beams: If the end portion of the beam extends beyond the support, such a beam is known as an overhanging beam.

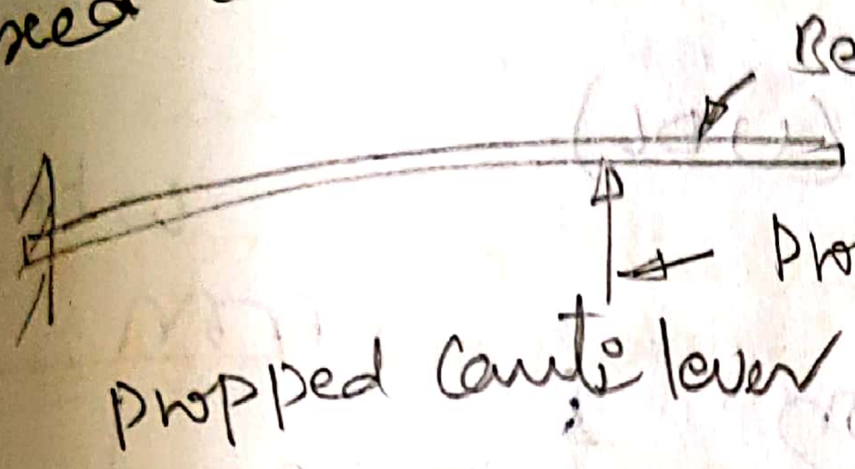


Ex: Railway sleepers, Bus bodies projection beyond wheel supports is a good example of

3. Cantilever Beam

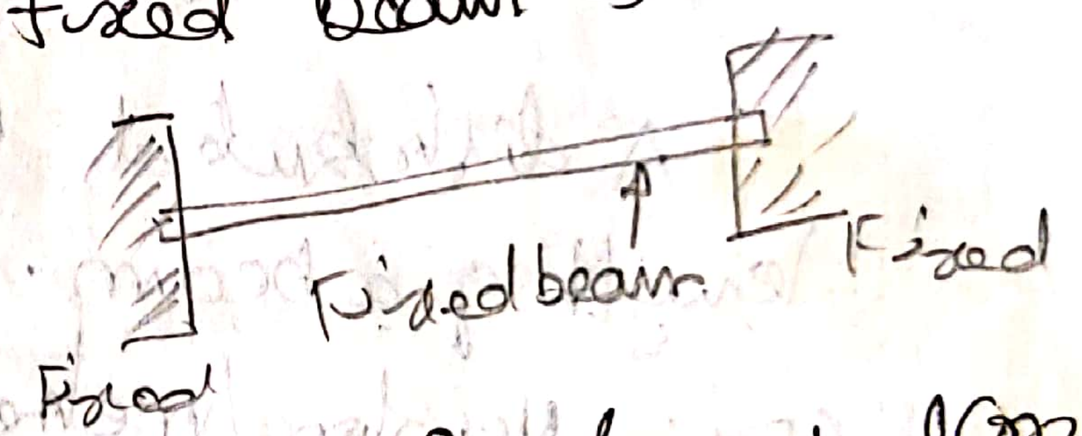


when one end of a beam is free and other end is fixed (in position and direction) the beam is called Cantilever beam.



Ex: portico, Telegraph post, sunshades etc.

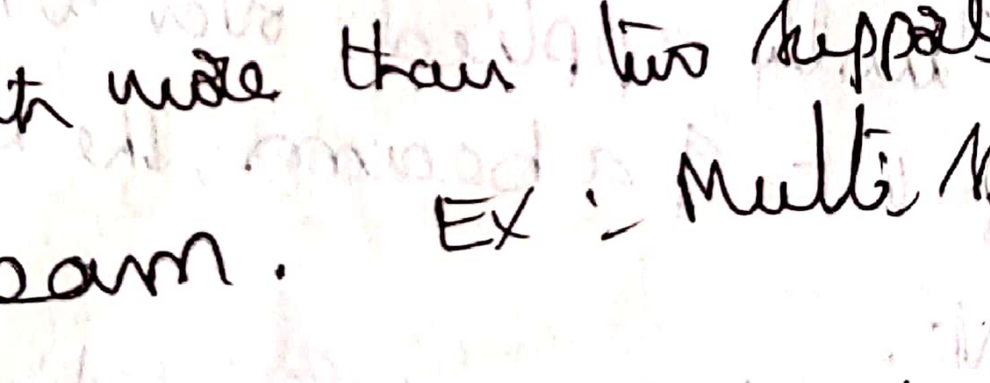
4. Fixed beams: A beam whose both ends are fixed or built-in is known as fixed beam or built in beam or encastred beam.



5. Continuous beam: A beam which is supported on more than two supports is known as a continuous beam. Ex:- Multi spanned bridges.

6. Propped Cantilever beam: As shown above.

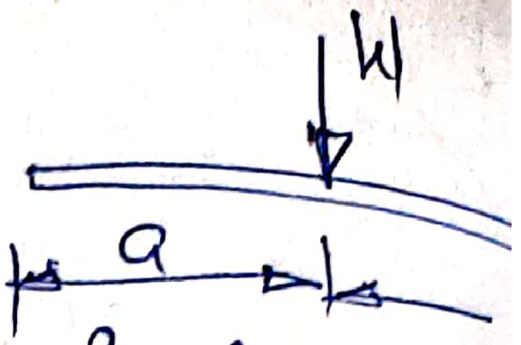
Note:



The clear horizontal distance b/t the walls — clear span
The Hor. distance b/t the centres of the end bearings - effective

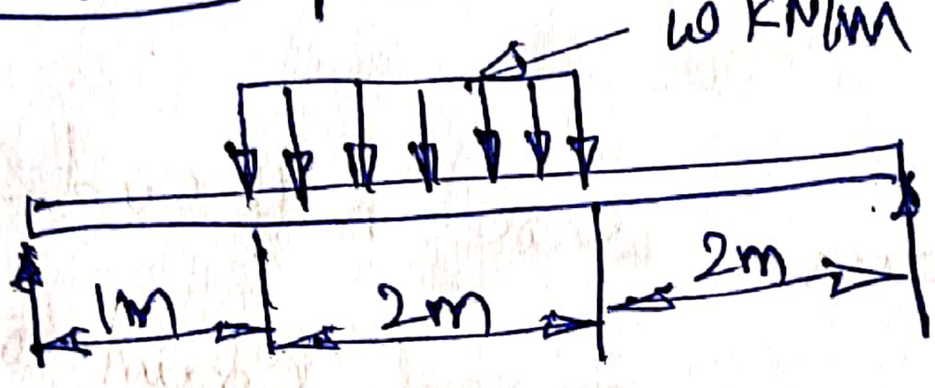
TYPES OF LOADS

1. Concentrated load (Point Load)

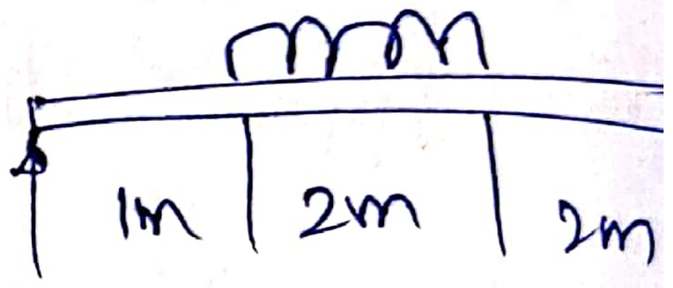


A Concentrated load is a load which is Conf'dered at a particular point. physically, loads Can be only over a finite area. However when the loads are Comparatively over a small area, it is Convenient of them as acting at a pt i.e Concentrated load wh resolved as the limit of the area of loading wh tends to zero.

2. Uniformly distributed load (UDL)



(or)



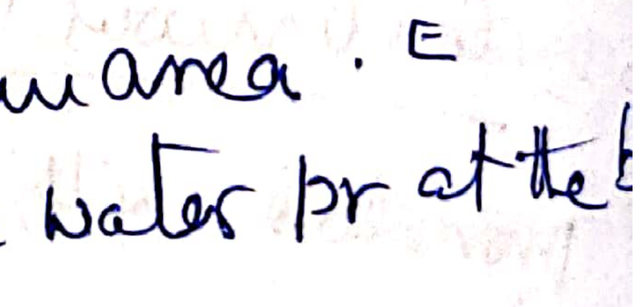
A distributed load is a load which is spread length of a beam. A dis. load is expressed by (Ex: Newton/metre).

If the intensity of the D.L is constant the load the load is actually applied over an area. E

Ex:: The self wt of a beam, the water pr at the b water tank.

It is represented in space diagrames thou and " as the magnitude of load / uni

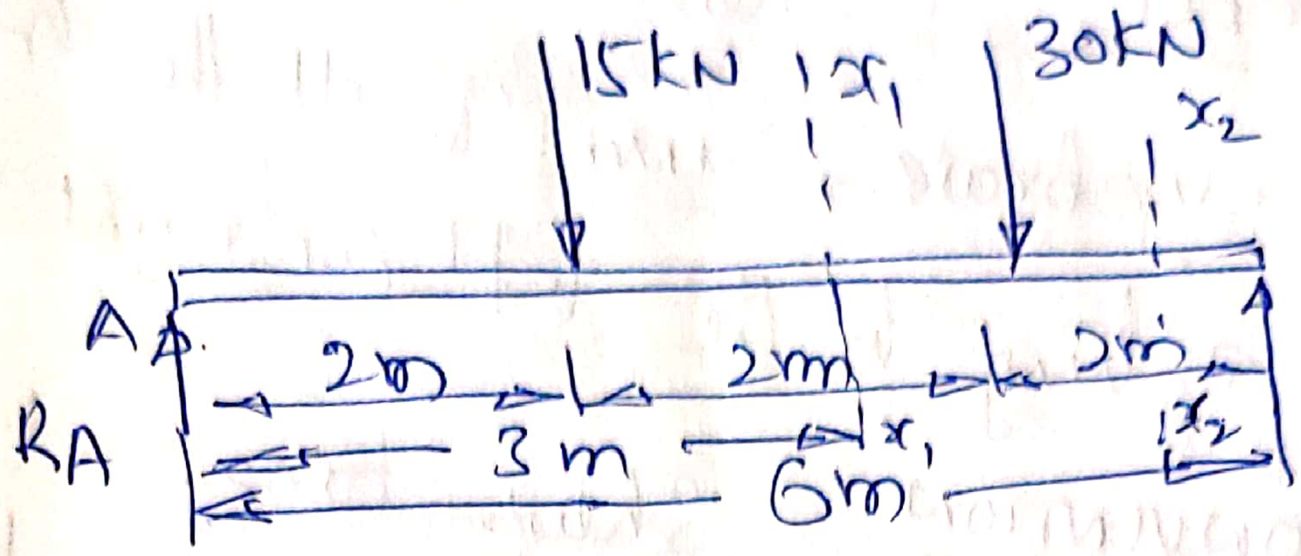
3. Uniformly Varying load



The intensity of loading varies uniformly from section to sect along the length of bea The water pressure on the vertical sides of water tank As shown the intensity of load is Zero @ left end @ right end.

SHEAR FORCE AND BENDINGMOMENT

Consider a simply supported beam AB of span 6m subjected to loads as shown.



$\Sigma M_B = 0$.

$R_A \times 6 - 15 \times 4 - 30 \times 2 = 0$

$R_A = \frac{60+60}{6}$

$= 20$ kN

$\therefore R_A + R_B = 15 + 30$

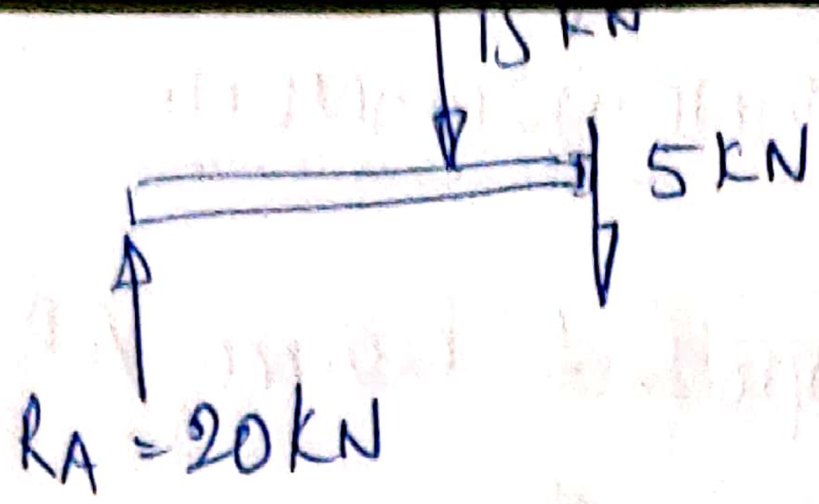
$\Rightarrow R_B = 25$ kN

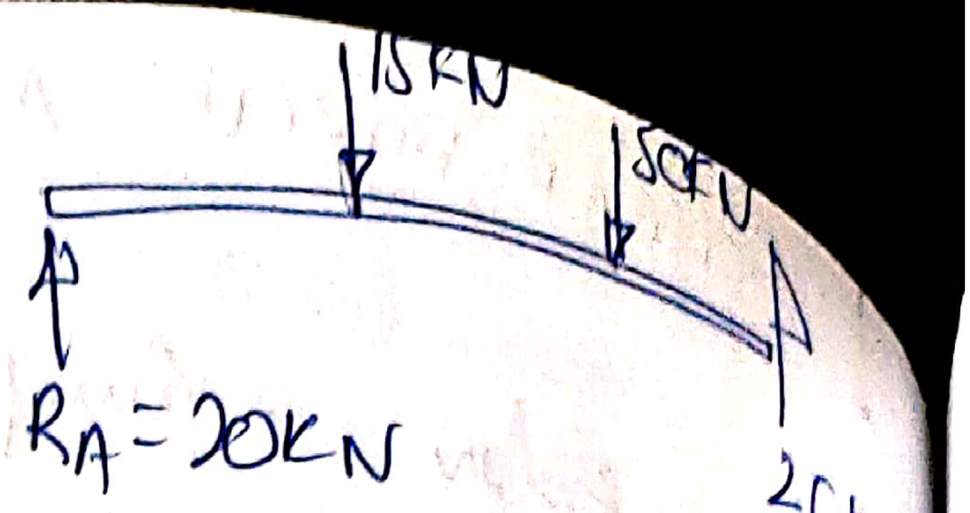
Reaction @ A $R_A = 20$ kN↑, $R_B = 25$ kN↑

Now Consider a section $x_1 x_1$ @ 3m from A as shown.

Treating the left portion of the beam as an isolated body, the vertical forces acting are reaction 20 kN acting upwards and external load 15 kN acting vertically downwards. To maintain equilibrium of the left portion of the isolated body, there must be an induced force 5 kN acting downwards at the cut section. This force is known as the shear force

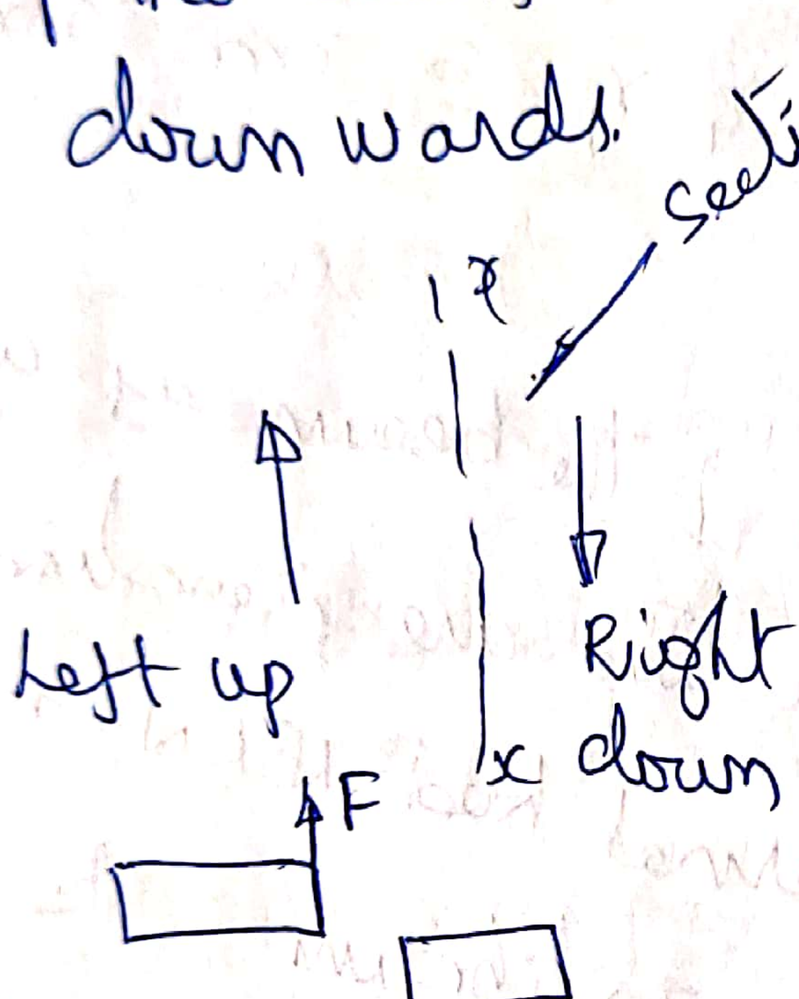
Based on similar arguments w.r.t. any section between B and load 30 kN, Considering the section $x_2 x_2$ b/t support B and load 30 kN, the shear force will be 25 kN upwards as shown.



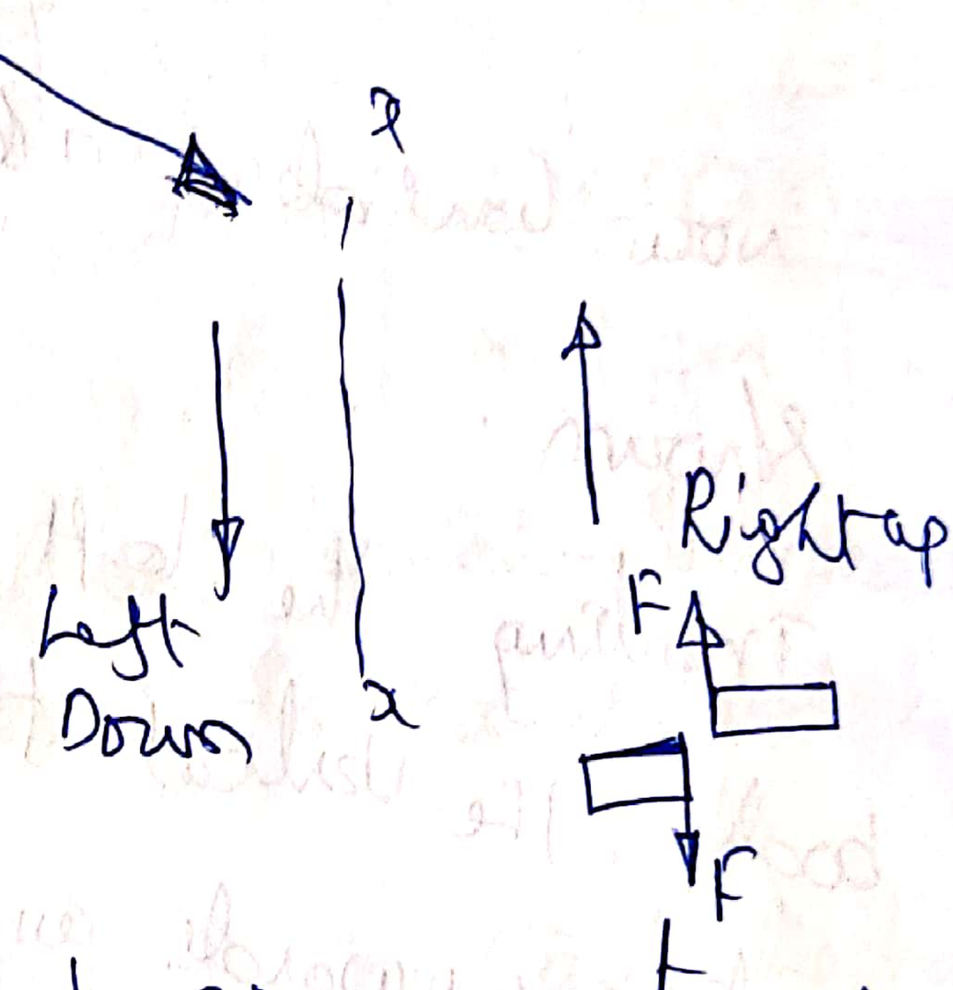


SHEAR FORCE: Shear force at any cross section is the algebraic sum of all the transverse forces acting to the left (or right) of the section.

SIGN CONVENTION: Shear force at a section is considered to be +ve when the resultant force to the left of the section is upwards (or) to the right of the section is downwards.



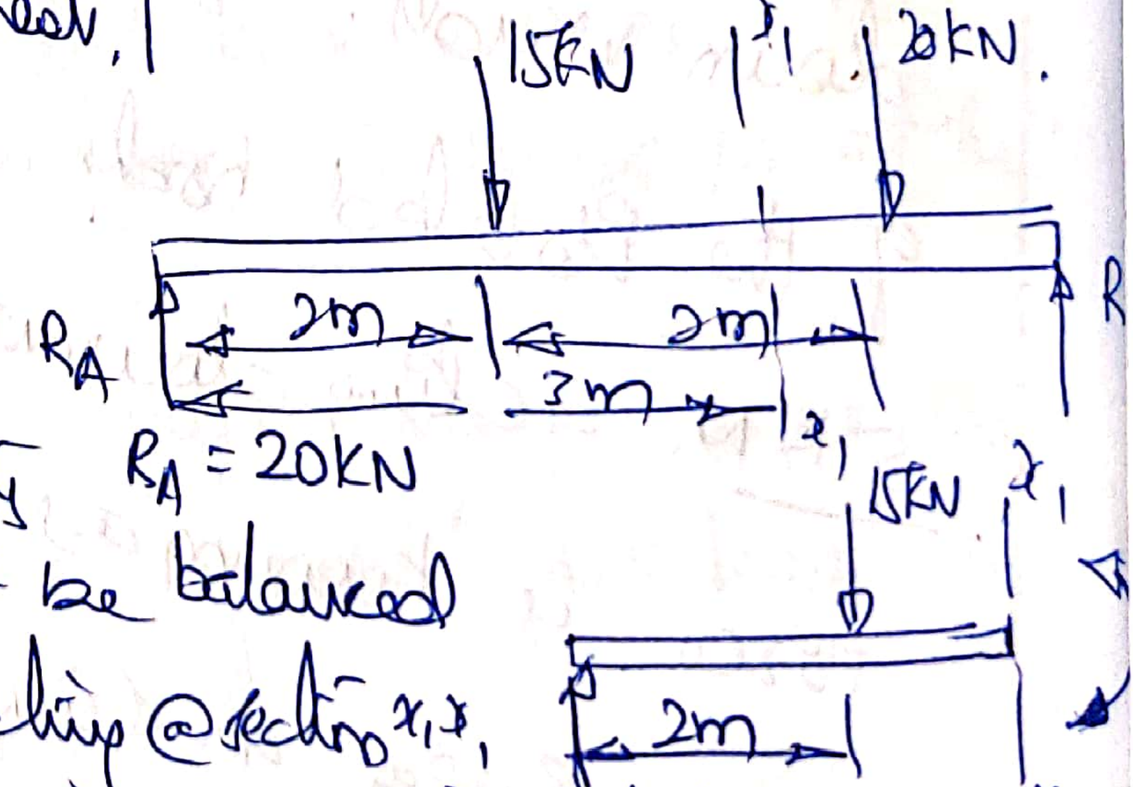
Sign Convention for +ve Shear.



Sign Convention for -ve

Bending moment

Resultant of all the moments acting in cut portion must be balanced by opposite moment M_x acting @ section x, x.



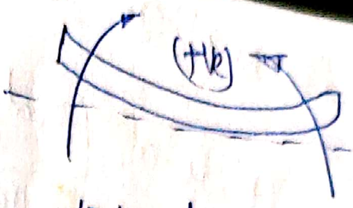
Def: ∴ Moment at any section (or point) of a beam which is equal to algebraic summation of moments of forces acting either to the right or to the left of the section is known as Bending moment.

SIGN Convention: for Bending Moment



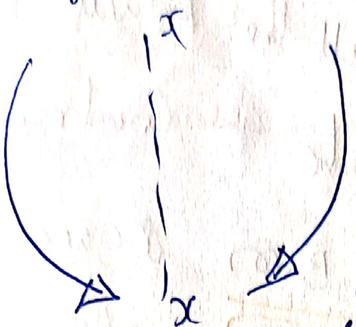
Left clockwise & Right anti clockwise

(a) Sign Convention for +ve bending moment.



Effect of Sagging B.M

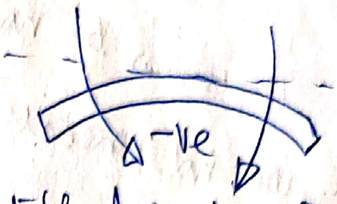
Concavity upwards



Left anti clockwise

Right clockwise.

(b) Sign Convention for -ve B.M



Convexity upwards

Effect of Hogging B.M.

Uses of B.M.D & S.F.D: Before taking up the design of any structural element, the structure is to be analysed and the magnitudes of bending moments & shear forces should be determined.

The structural element is designed for more B.M & for max. shear force.

The B.M diagrams help to a great extent in identifying the tensile zones in reinforced Con crete structures for providing steel reinforcement at appropriate places.

S.F.D & B.M.D are very much used in design offices.

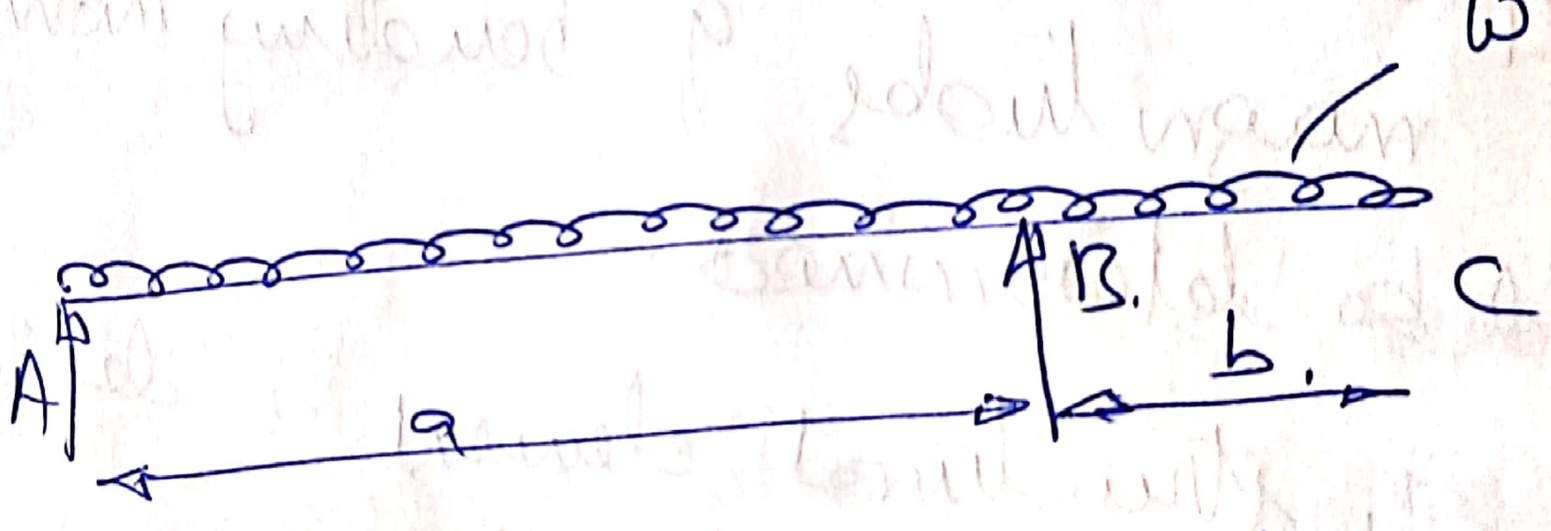
POINT OF CONTRAFLEXURE (POINT OF

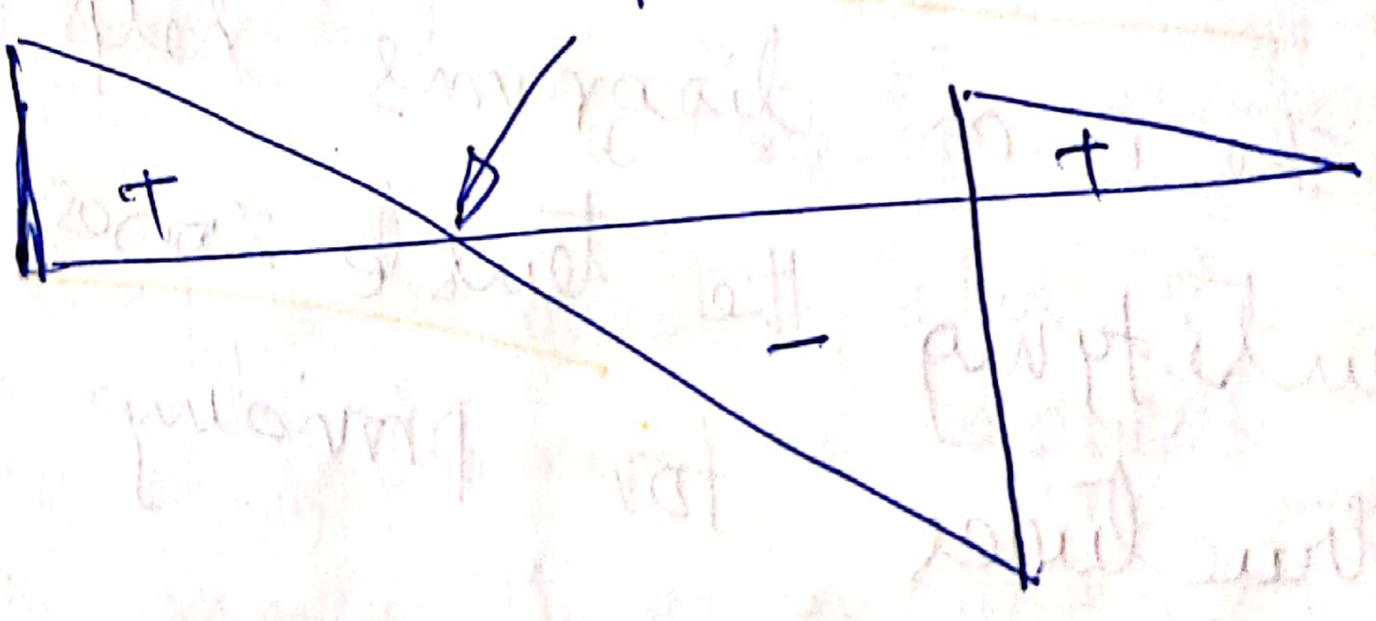
It is a point where the bending moment changes from +ve to -ve (or) the bending changes from to hogging and vice-versa.

At the point of Contraflexure, the B.M is

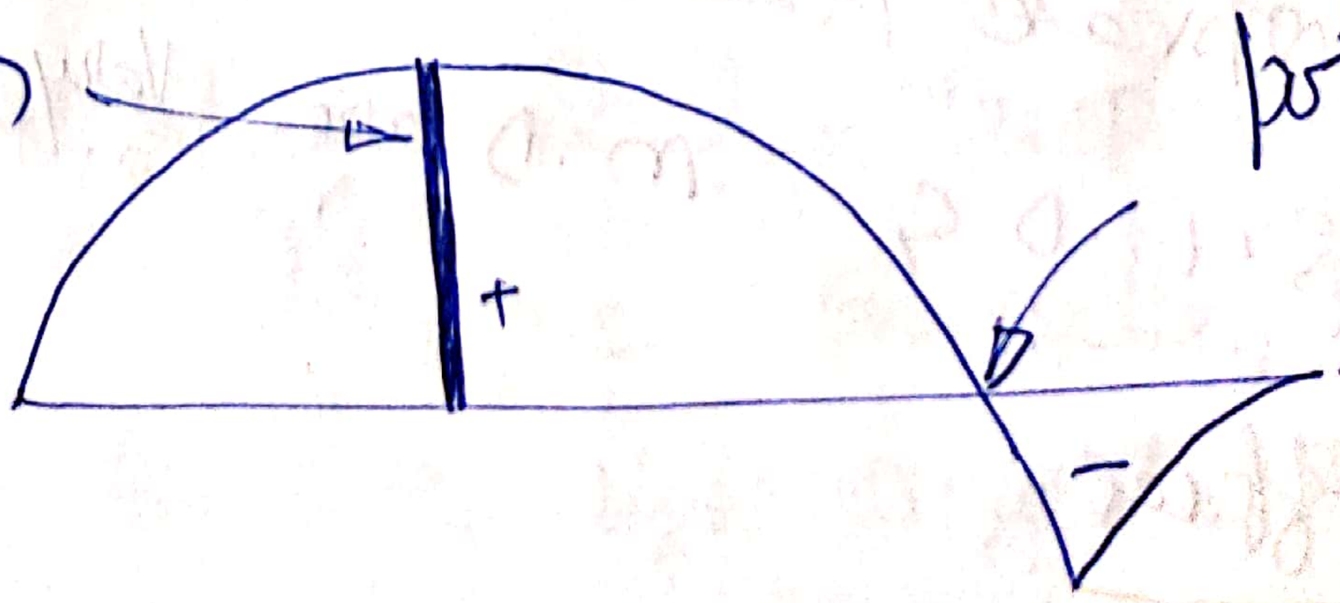
It is to be noted that all the points where B.M are not necessarily points of Contraflexure.

Generally, max. B.M occurs at the where the shear force is zero. This point should be remembered while drawing the B.M.D and to that the existence of a proper Compatibility B.M.D & S.F.D can be observed.





S.F.D.

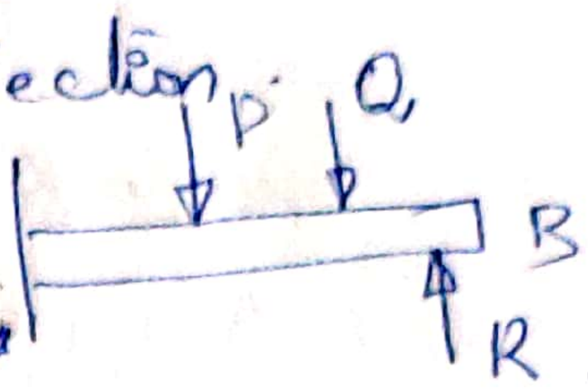


B.M.D

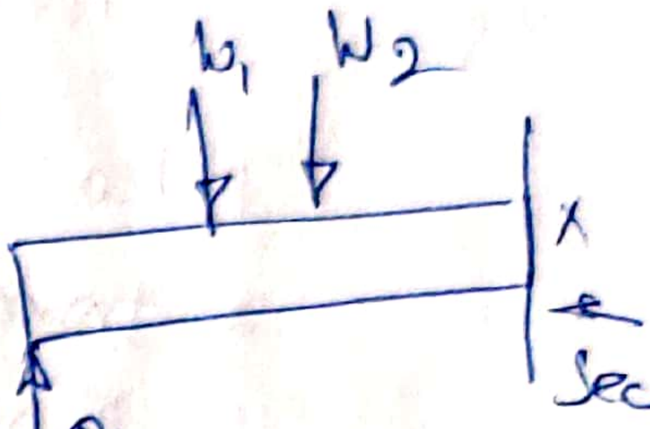
IMPORTANT

SHEAR FORCE: To Calculate the S.F @ a section consider left (or) right part of the section

i) If right part of the section is chosen a force on the RIGHT part acting downwards is +ve while the force on right part is -ve upwards is -ve. ∴ S.F @ section x of a beam is required, if xB i.e right part of section is considered S.F @ x = P+Q-R.



Similarly . S.F @ x = Q-W1-W2

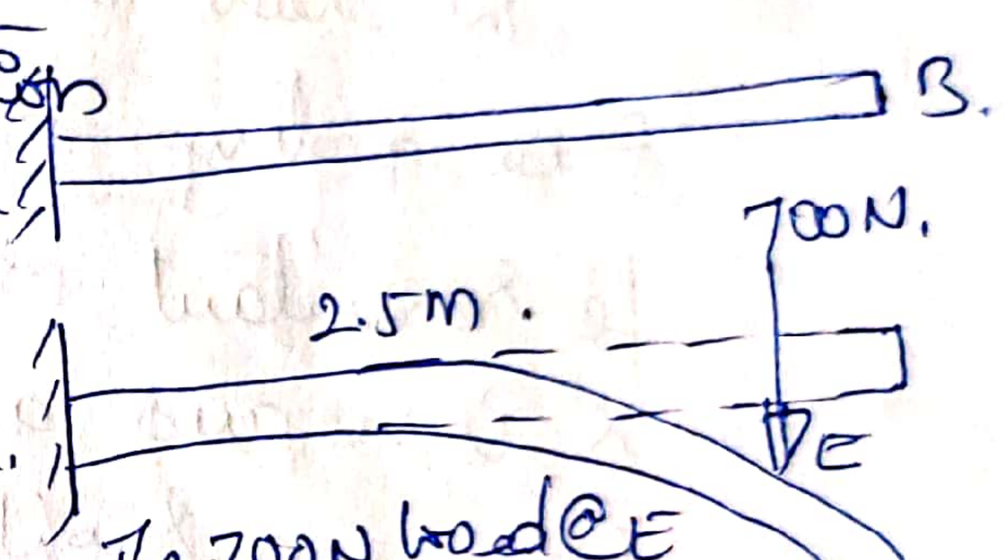


BENDING MOMENT: To find B.M @ a section

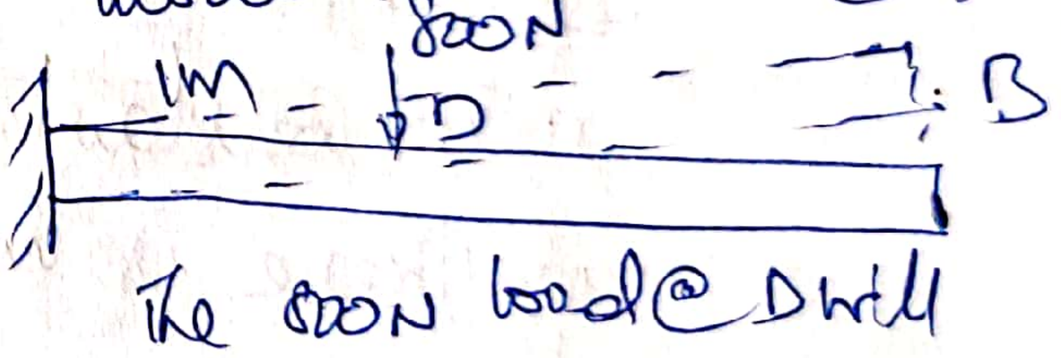
i) Consider left (or) right part of section

ii) Remove all restraints and all forces on the part selected

iii) Now Introduce each force (or) reaction element one at a time and find its effect at the section (Find whether the moment produced a hogging (or) sagging effect at the section)



The 700N load @ E will produce a hogging moment of 1750N-m @ A.



The 800N load @ D will produce a hogging moment at 800N-m @ A.

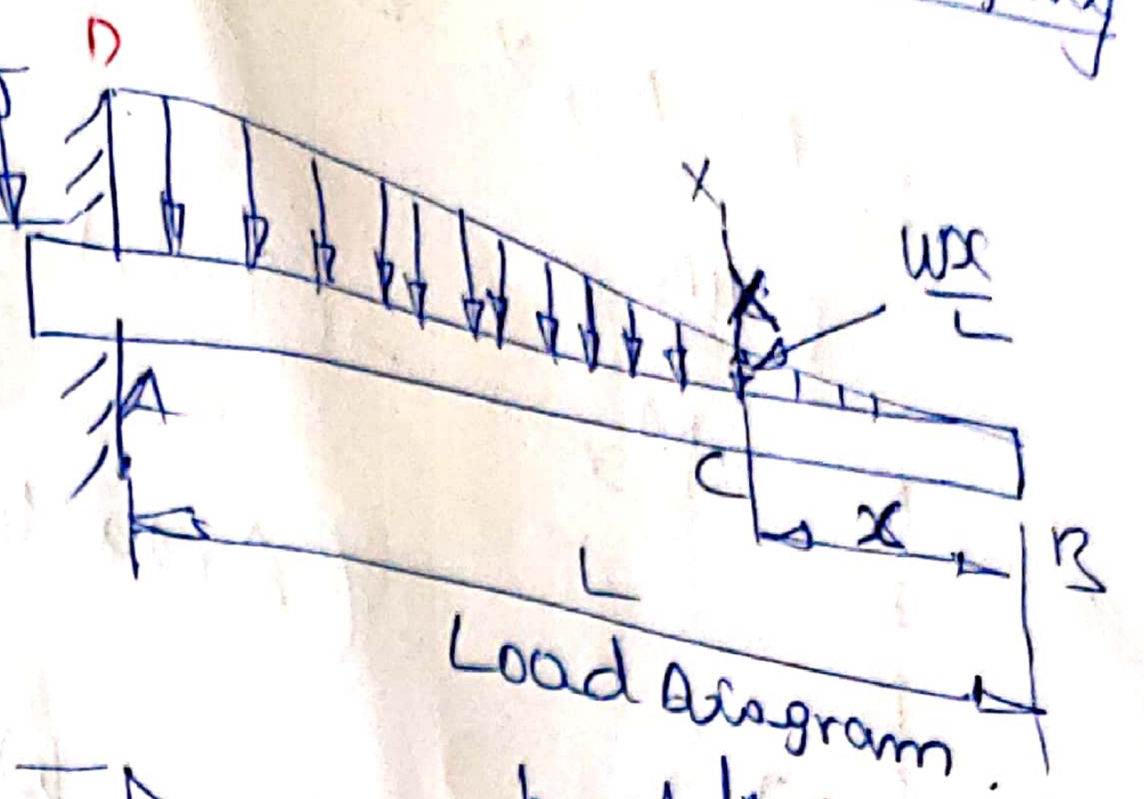
Note that the moment due to every downward force is -ve and the moment of due to every upward force is +ve.

If right part of section is selected. Remove restraint on part AB Introduce the load of 700N @ E The independent effect of the load is to produce a hogging moment of $-700 \times 2.5 = -1750$ N-m. Similarly of 800N $-800 \times 1 = -800$ N-m

∴ Resultant B.M @ A = -1750-800 = -2550 N-m

S.F. & B.M. diagrams for a cantilever carrying a gradually Varying Load

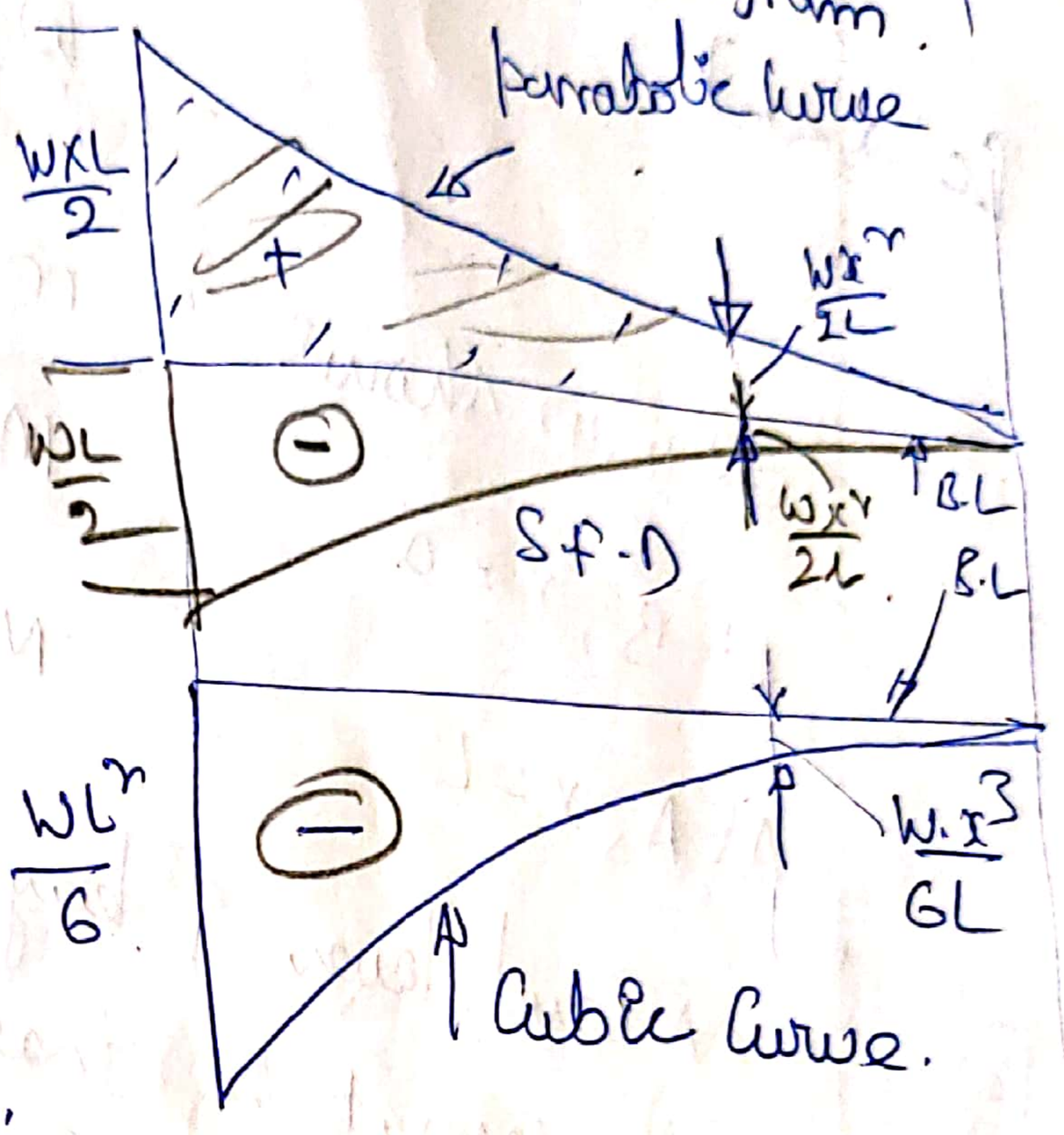
Load diagram shows a Cantilever of Length L fixed at A and carrying a gradually varying load from zero at the free end to w per unit length at fixed end.



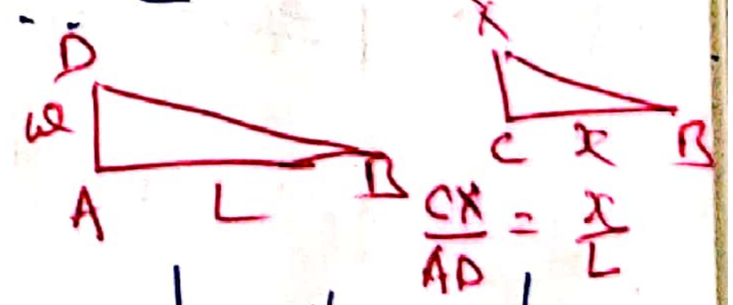
Take a section X @ a x from free end B.

Let F_x = Shear force @ the section x &

M_x = Bending moment @ the section x.



Rate of loading at the section x: The rate of loading is zero at B and is w per metre run at A. This means that rate of loading for a Length L is w per unit length. Hence rate of loading for a length of x will be $\frac{w}{L} \times x$ per unit length. ∴ C



$$CX = \frac{w.x.}{L}$$

∴ F_x = Total load on the Cantilever for a length x from the free end B = Area of Δle BCX

$$= \frac{BC \times CX.}{2} = \frac{x.\frac{wx}{L}.}{2} = \frac{wx^2}{2L} \quad -(i)$$

∵ BC = x, $CX = \frac{w.x}{L}$

Eq (i) Varies according to the parabolic law.

At B, $x = 0$, $F_B = \frac{w \cdot 0}{2L} = 0$

At A, $x = L$ $F_A = \frac{w \cdot L^2}{2L} = \frac{w \cdot L}{2}$

B.M:

$M_x = -$(Total load for a length x) x Distance from x x

$= -$(Area of Δle BCX) x Distance of C.G from x x

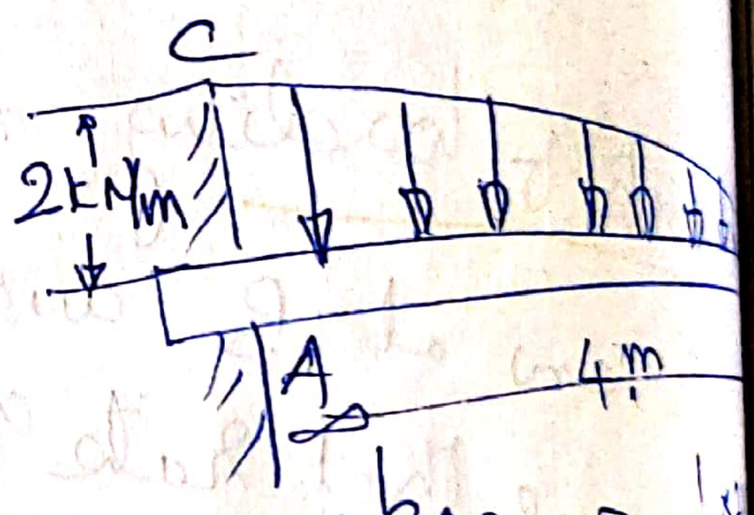
$[0 < x < L]$ $= -\left(\frac{w x^2}{2L}\right) \times \frac{x}{3} = -\frac{w x^3}{6L}$.

Q11 (ii) Show B.M varies according cubic

At B, $x = 0$ $M_B = \frac{-b \times 0}{6L} = 0$

At A, $x = L$ $M_A = -\frac{w \cdot L^3}{6L} = -\frac{w \cdot L^2}{6}$.

(Pr). A Cantilever of length 4m carries a gradually load, zero at the free end to 2 kN/m at fixed Draw the S.F & B.M diagrams for Cantilever.



Sol: Length $L = 4$ m

Load @ fixed end $w = 2$ kN/m.

S.F.D: $F_B = 0$, $F_C =$ Area of $\Delta^{le} ABC = \frac{1}{2} \times$

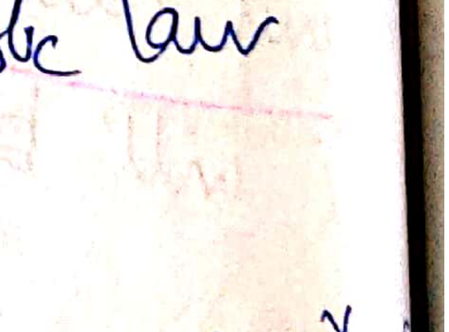
S.F varies according to parabolic law

B.M.D: The B.M at B is zero.

$M_B = -\frac{wL^2}{6} = -\frac{2 \times 4^2}{6} =$

$M_A =$.

The B.M Between A & B varies according to



$M_x = \frac{wb}{L} x - w(x-a)$

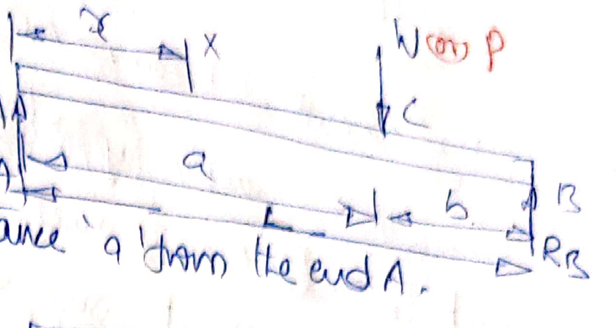
@ C, $x = a$ $M_C = \frac{wba}{L} - w(a-a) = -\frac{wab}{L}$

@ B, $x = L$, $M_B = \frac{wb}{L} \times L - w(L-a)$

[$\because L - a = b$]

S.F.D & B.M.D for a Simply Supported Beam with an Eccentric Point Load

Fig: shows a beam AB of length L, simply supported at the ends A & B & carrying a point load W @ C at a distance 'a' from the end A.



Let R_A = Reaction @ support A

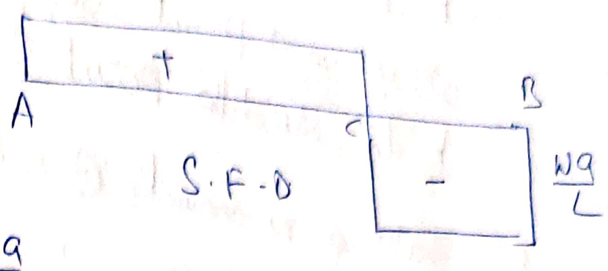
R_B = " " B

Taking moment about A (or) B

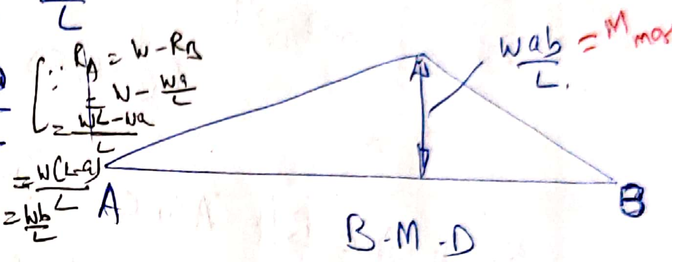
$R_B \times L = W \times a \Rightarrow R_B = \frac{Wa}{L}$

$R_A + R_B = W \Rightarrow R_A = \frac{Wb}{L}$ [$\because R_A = W - R_B = W - \frac{Wa}{L} = \frac{WL - Wa}{L} = \frac{W(L-a)}{L} = \frac{Wb}{L}$

$\because L - a = b$]



S.F.D



B.M.D

S.F.D: $F_x = +R_A = +\frac{Wb}{L}$

Shearforce $F_{A-C} = \frac{Wb}{L}$.

Now Consider any section b/t C & B at a distance x from the end A. The resultant force on the left portion will be $R_A - W$

$$\frac{Wb}{L} - W = W\left(\frac{b-L}{L}\right) = -W\left(\frac{L-b}{L}\right) = -\frac{Wa}{L} \quad [\because L - b = a]$$

$\therefore F_{CB} = -\frac{Wa}{L}$. S.F changes from $\frac{Wb}{L}$ to $-\frac{Wa}{L}$ @ C

B.M.D: Between A & C, $M_x = R_A \cdot x = +\frac{Wb}{L} \cdot x$.

(+ due to sagging) $[0 < x < a]$

At A, $x = 0$, $M_A = 0$.

At C, $x = a$, $M_C = \frac{Wb}{L} \cdot a = \frac{Wab}{L}$.

$\therefore$ B.M increases from Zero at A to $\frac{Wab}{L}$ @ C. by St. Law.

$\therefore$ B.M decreases from $\frac{Wab}{L}$ @ C to Zero at B. following Straight Line Law.

B.M @ B = 0

B/t C & B, $M_x = R_B x = \frac{Wax}{L}$

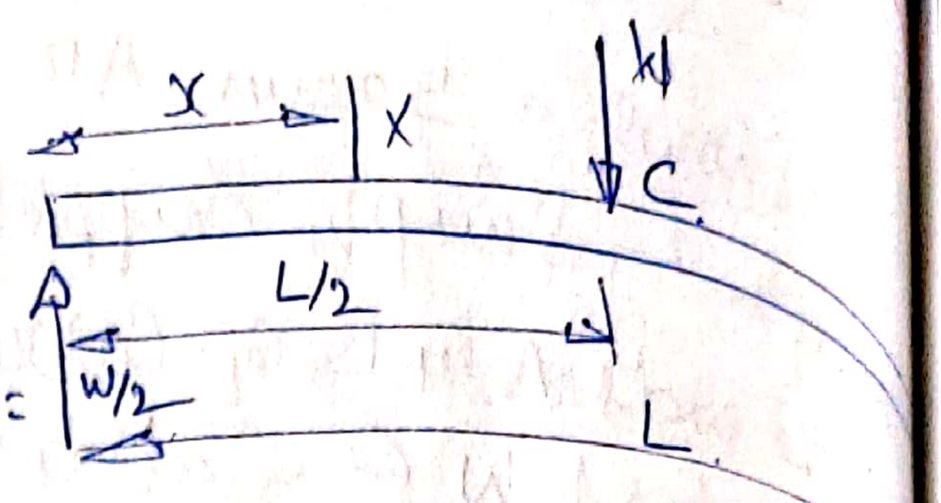
@ C $M_C = \frac{Wa}{L} b = \frac{Wab}{L}$

$B.M_{max} = \frac{Wab}{L}$ (@ C)

$= \frac{Pab}{L}$

SHEAR FORCE AND B.M.D For a Simply Supported Beam with a point load at Mid ~~Point~~ of Span

Fig shows a beam AB of length L Simply supported at the ends A & B and carry a point Load W at its middle point C



Taking moments @ A

$$W \times \frac{L}{2} + R_B \times L = 0$$

$$W \frac{L}{2} = -R_B \times L \Rightarrow R_B = \frac{W}{2}, \quad R_A = W/2$$

Taking moments @ B

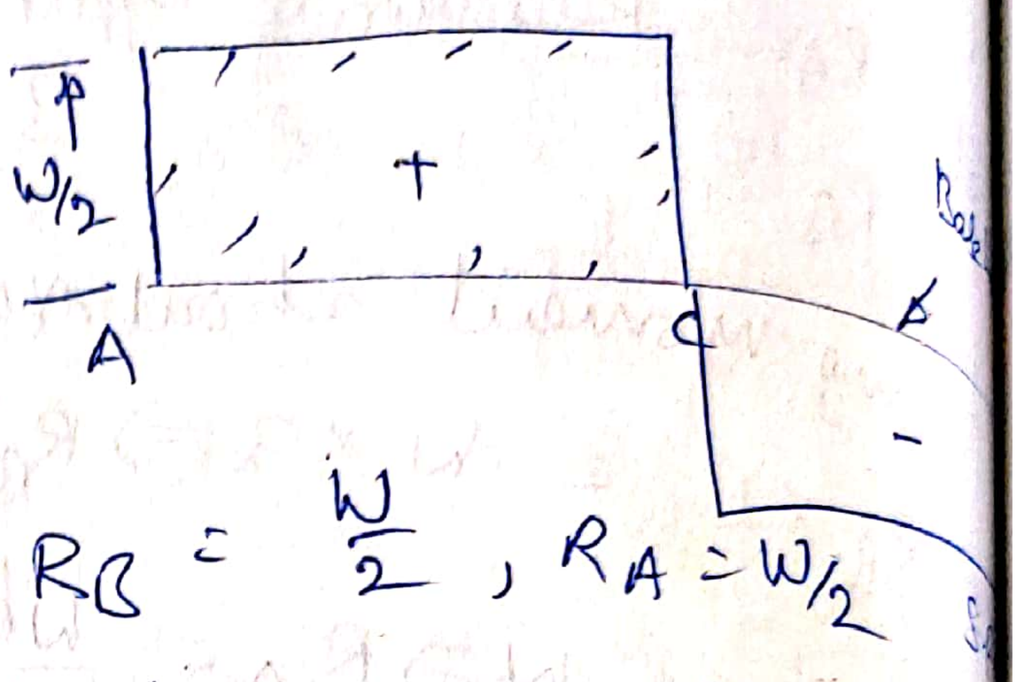
$R_A \times L = W \times \frac{L}{2}$

$R_A = \frac{W}{2}$

$R_A + R_B = W$

$R_B = W - \frac{W}{2}$

$= \frac{W}{2}$



Shear Force B/t A & C $= +\frac{W}{2}$

Consider any section B/t C & B at distance x from end A

The resultant force $\frac{W}{2} - W = -\frac{W}{2}$

This force will also remain constant b/c C & B.

S.F b/t C & B is equal to $-W/2$

At section C the S.F. changes from $+\frac{W}{2}$ to $-\frac{W}{2}$

B.M will be Max where S.F changes its sign from +ve to -ve

B.M.D: (i) $M_x = R_A \cdot x$ (or) $M_x = \frac{W}{2} \cdot x$. $[0 < x <$

At A, $x = 0$, $M_A = \frac{W}{2} \times 0 = 0$

At C, $x = \frac{L}{2}$, $M_C = \frac{W}{2} \times \frac{L}{2} = \frac{WL}{4}$

B.M varies according Straight line Law from A & C

(ii) B/t C & B at x from the end A is

$$M_x = R_A \cdot x - W\left(x - \frac{L}{2}\right)$$

$$= \frac{W}{2} x - Wx + \frac{WL}{2} = \frac{WL}{2} - \frac{Wx}{2}$$

At C, $x = \frac{L}{2}$ $M_C = \frac{WL}{2} - \frac{W}{2}\frac{L}{2} = \frac{WL}{4}$.

At B, $x = L$ $M_B = \frac{WL}{2} - \frac{WL}{2} = 0$.

UNIT IV

SHEARING STRESSES IN BEAMS

Beams are always s.t shearing stresses (Forces) if the B.M is not constant at every C/S.

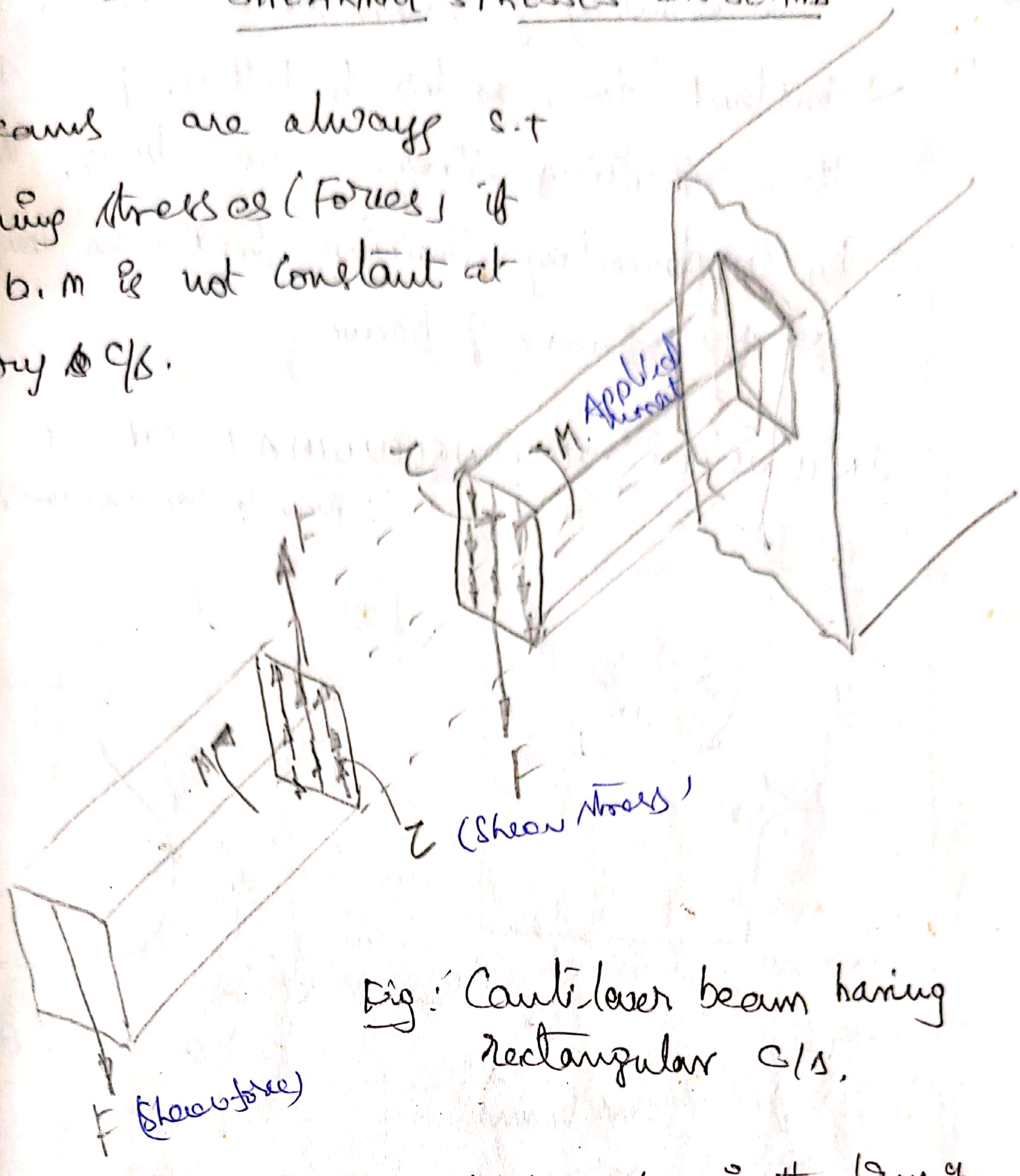


Fig: Cantilever beam having rectangular C/S.

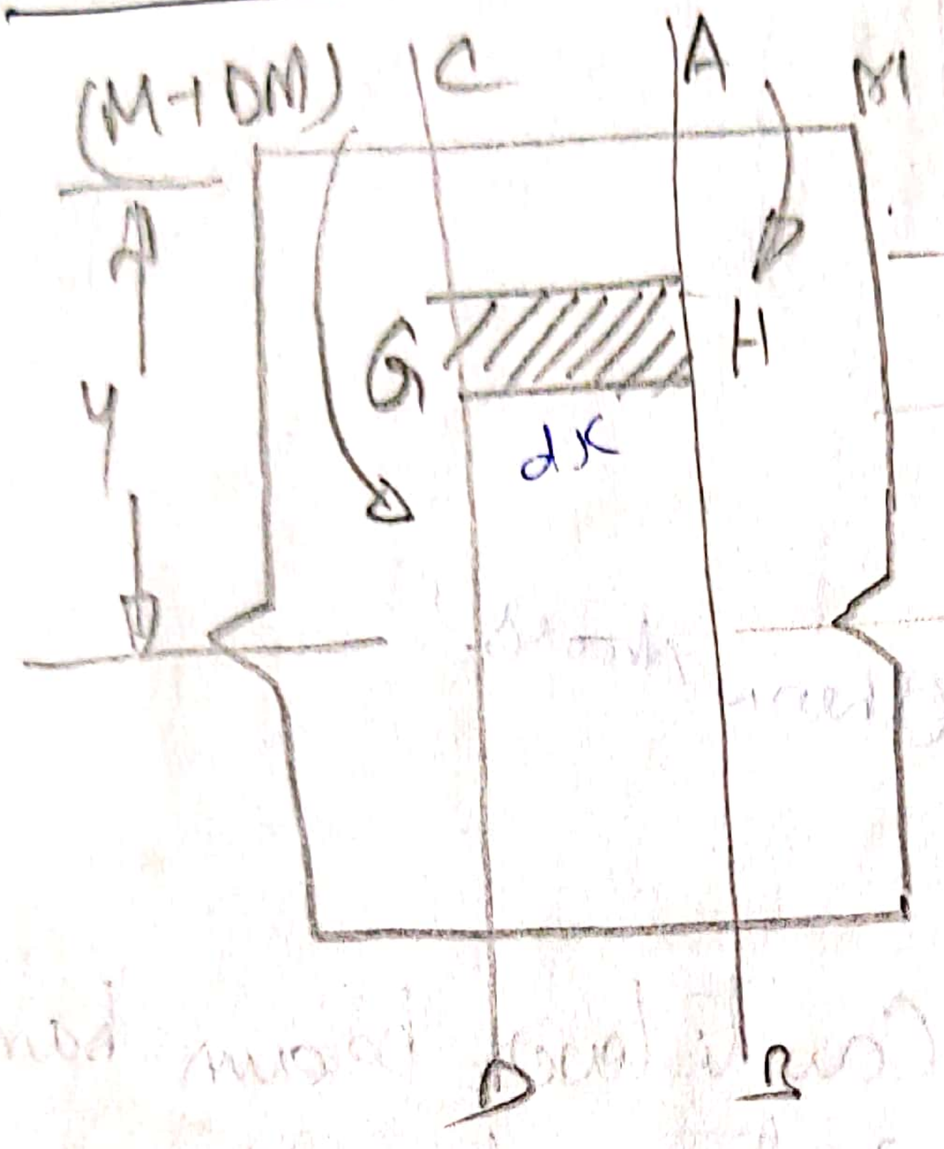
⇒ Consider the effect of shear force in the form of a shearing stress acting tangentially to the C/S of the beam.

⇒ If the beam is cut any transverse section, to maintain equilibrium the B.M & S.F's F are to be applied at that section.

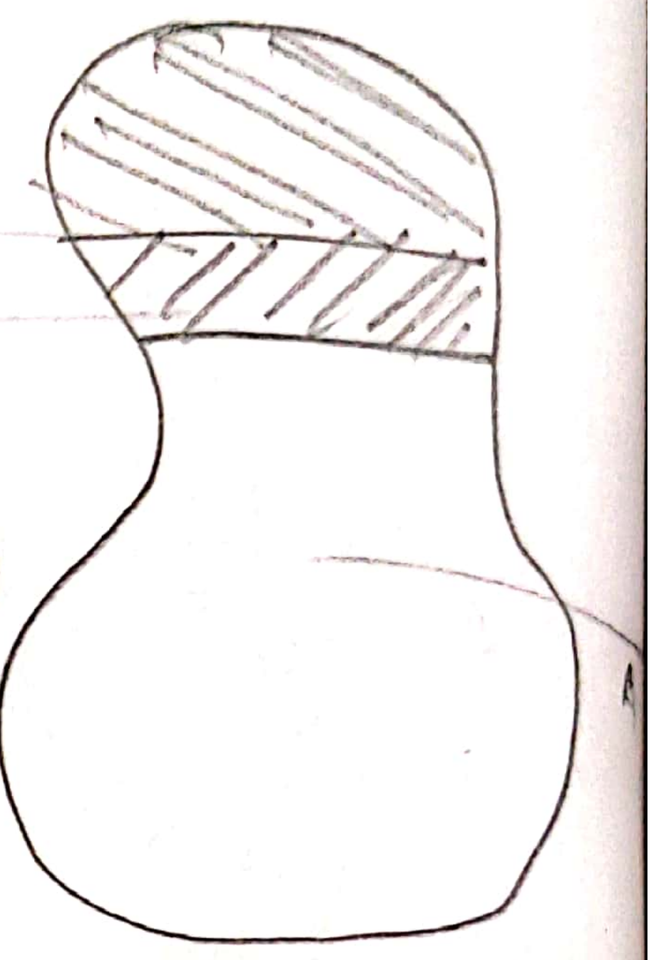
⇒ The B.M is distributed across the depth in the form of a longitudinal direct stress $\boxed{\sigma = \frac{M}{I} y}$ & the shearing force F is distributed in the form of a shearing stress τ acting tangentially to C/S of a beam.

⇒ The intensity of shear stress τ on the section is constant from the top to bottom of the section. These shearing stresses are always accompanied by complementary shearing stresses in longitudinal to the axis of beam.

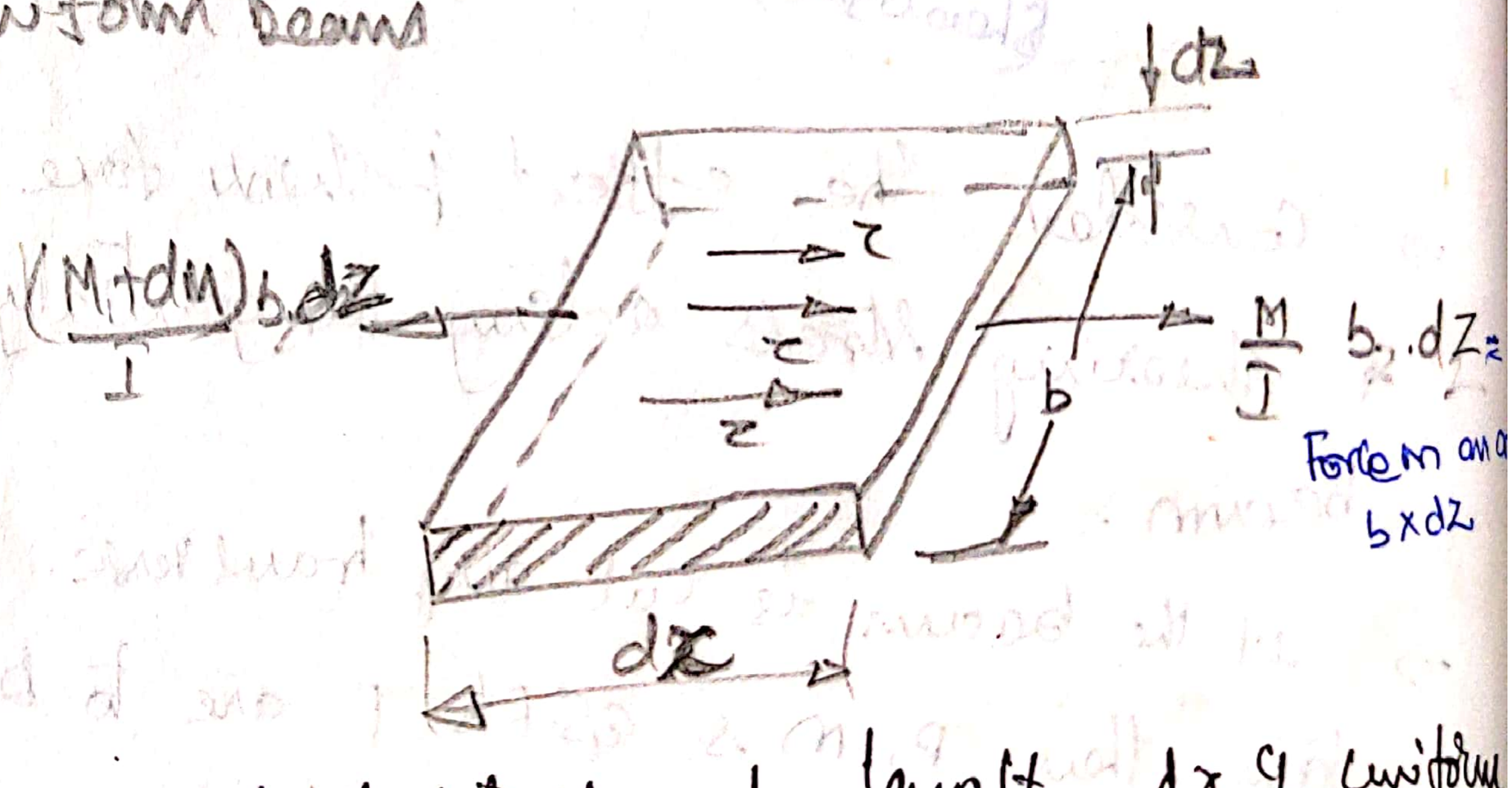
INTENSITY OF LONGITUDINAL STRESSES : Derivation Formula for shear stress



(a) I-Section uniform beams



(b) C/S of beam.



⇒ Consider an infinitesimal length dz of uniform

⇒ Let the B.M acting at the vertical faces AB & CD be M & M+dM respectively.

⇒ Let σ be the intensity of stress at 'H' due to B.M on a small area & width b & thickness dz & σ_1 be the stress at 'G' on a corresponding area of C/S.

Using simple Bending formula

Stress $\sigma = \frac{M}{I} z$, $\sigma_1 = \frac{M+dM}{I} z$. [$\because y = z$]

$\therefore$ Force on an area $b\,dz$ due to Bm M =

= Stress × Area = $\sigma (b\,dz) = \left(\frac{M}{I} z\right) b\,dz$

Force on an area $b\,dz$ due to $M+dM$

$= \left(\frac{M+dM}{I} z\right) . b\,dz.$

$\therefore$ Resultant force on a small piece of length dx, width b & thickness dz.

$= \left(\frac{M+dM}{I} z\right) b\,dz - \frac{M}{I} z\, b\,dz$

$= \boxed{\frac{dM}{I} b z\, dz.}$

Total Resultant force pull on length dx and area shown as I

$= \int_z^y \frac{dM}{I} b z\, dz.$ [apply to total area in the limits of z to y]

This resultant pull on the piece is resisted by shearing force on the left longitudinal section at GH whose area is $(b \cdot dx)$ [Note: shear stress acts tangential to surface]

Let z = Intensity of shear stress on this area

$\therefore z(b \cdot dx) = \frac{dM}{I} \int_z^y b z \cdot dz.$ [$\because$ shear force = shear stress × area = $z(b\,dx)$]

$\therefore z = \frac{dM}{dx} \frac{1}{Ib} \int_z^y b z\, dz.$

But $\frac{dM}{dx}$ = Shear Force F.

and $\int (b\,dz)\, z$ = Sum of the moments of areas of all the small strips comprising the shaded area about N.A.

If a is the moment area about the plane, whose centroid and $\bar{y}$ is the distance of its centroid from N.A, then

$$\int_z^y (b\,dz)z = a\bar{y}$$

$$\therefore \boxed{\tau = \frac{F a \bar{y}}{I b}}$$

where F = shear force at the section

Shearing stress at a section $\tau = \frac{F a \bar{y}}{I b}$

Note 1. For any given C/S and loadings, the distribution of τ will vary directly with the magnitude of $\bar{y}$ since F, a, I are constants.

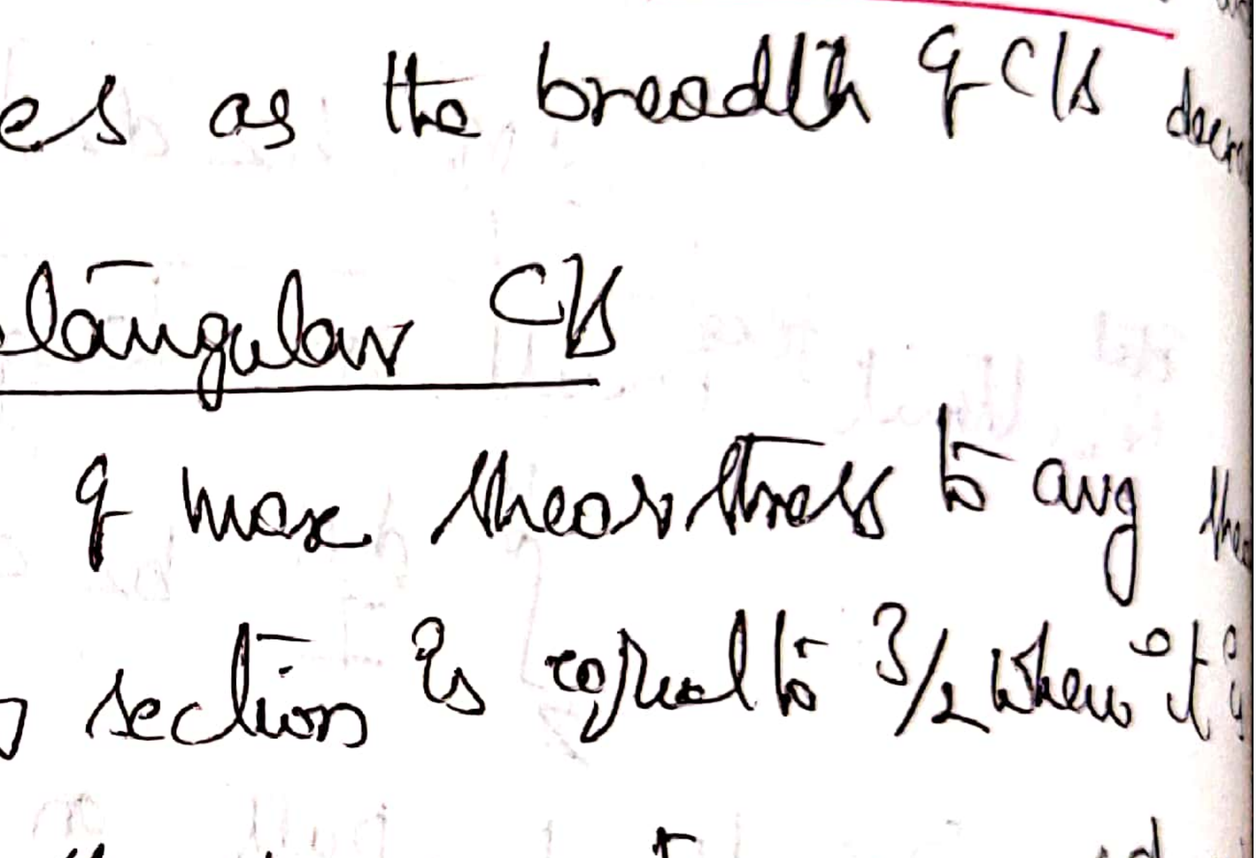
2. The shearing stress will be zero at the top and bottom of any CS since $a\bar{y}$ will be zero and its value increases as the breadth of CS decreases.

I Shear Stress For Rectangular CS

Show that the ratio of max shear stress to avg shear stress in a rectangular section is equal to 3/2 when it is S.T to a transverse shearing force F.

Sol: Consider a rectangular section of breadth 'b' & depth d.

Consider a section XX at a distance of y from N.A.



Shear stress intensity at this level $\tau_{xx} = \frac{F a \bar{y}}{I b}$

where a = area of C/S above XX = $b\left(\frac{d}{2} - y\right)$

and $\bar{y}$ = the distance of C.G. from N.A. [1 of shaded area ABCD]

$= y + \frac{1}{2}\left(\frac{d}{2} - y\right) = y + \frac{d}{4} - \frac{y}{2}$

$= \frac{1}{2}\left(\frac{d}{2} + y\right)$

$\therefore \tau_{xx} = \frac{F}{2 \times \frac{bd^3}{12}} \left[\frac{d^2}{4} - y^2\right] = \frac{6F}{bd^3}\left[\frac{d^2}{4} - y^2\right]$ *

At the top edge $y = +\frac{d}{2}$, $\tau = 0$

At the bottom edge $y = -\frac{d}{2}$, $\tau = 0$.

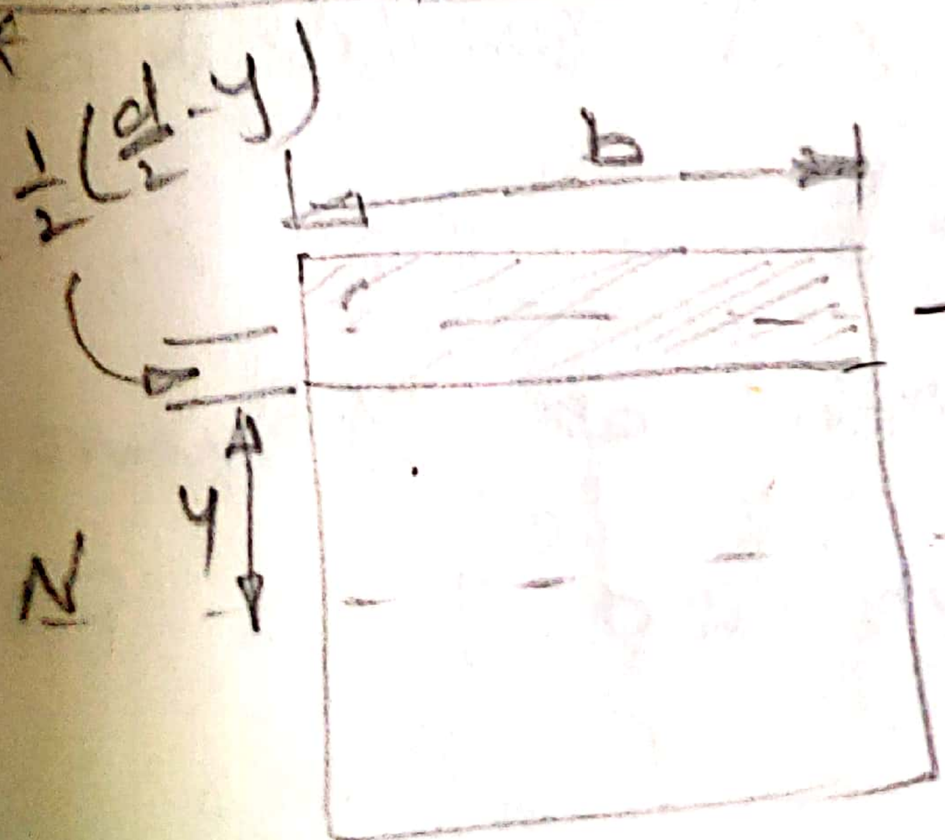
At N.A $y = 0$, $\tau = \frac{3}{2}\frac{F}{bd}$. ∵ Degree of y is 2

The variation of shear stress is parabolic and the max. shear stress occurs @ N.A.

Avg shear stress $= \frac{F}{bd}$ [Stress = F/Area]

$\therefore \frac{\tau_{max}}{\tau_{avg}} = \frac{3/2\ F/bd}{F/bd} = \frac{3}{2}$ ✓

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* 

$\frac{1}{2}\left(\frac{d}{2} - y\right)$

$\therefore \bar{y} = y + \frac{1}{2}\left(\frac{d}{2} - y\right) = y + \frac{d}{4} - \frac{y}{2}$
$= \frac{1}{2}\left[\frac{d}{2} + y\right]$

$\bar{y}$ = Moment of area above xx about N.A

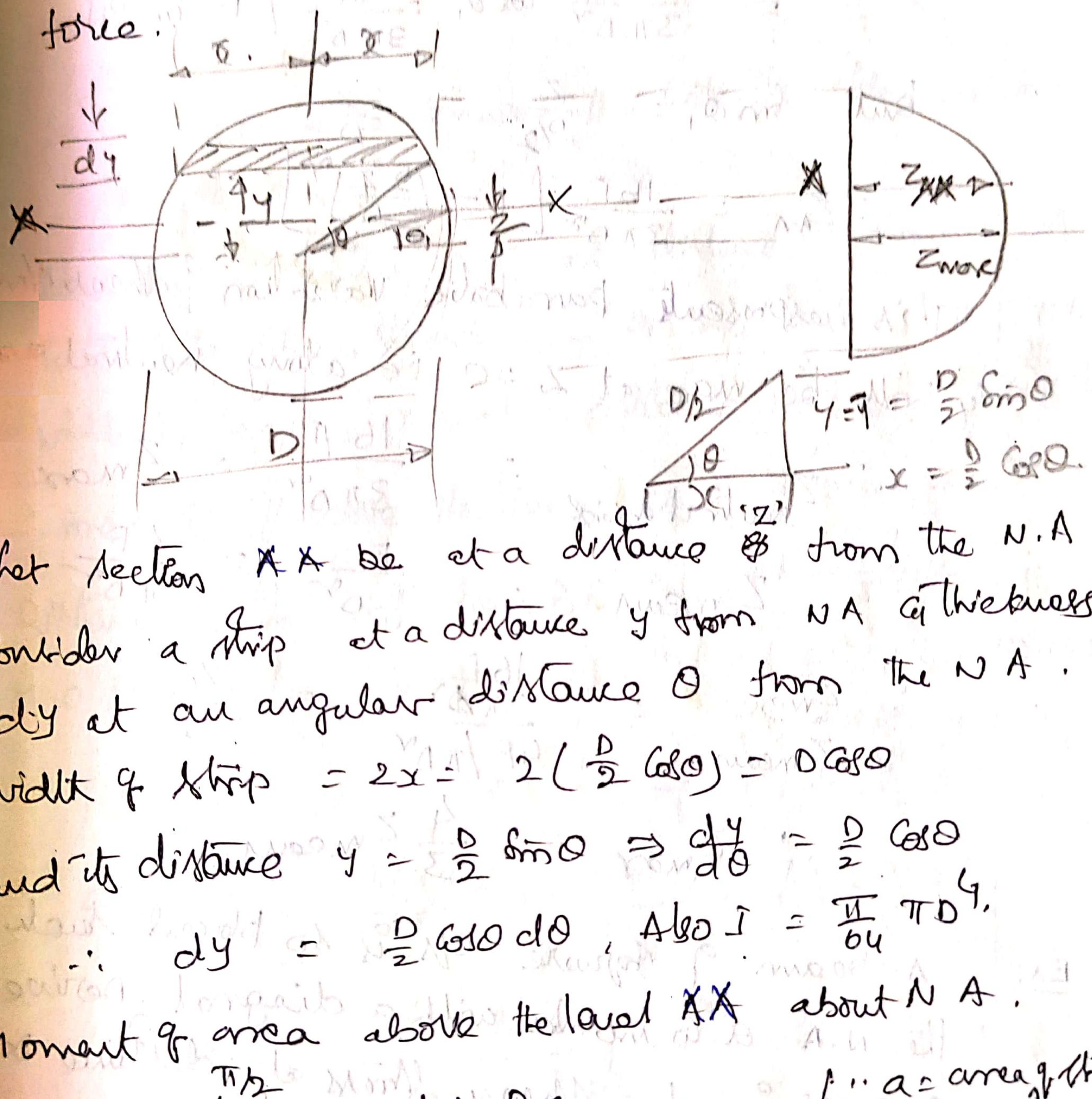
$\tau_{xx} = \frac{F a \bar{y}}{I b}$

$I = \frac{bd^3}{12}$, $a = b\left(\frac{d}{2} - y\right)$, $\bar{y} = \frac{1}{2}\left(\frac{d}{2} + y\right)$

$\therefore \tau_{xx} = \frac{F \cdot b\left(\frac{d}{2} - y\right) \times 1 \left(\frac{d}{2} + y\right)}{\frac{bd^3}{12} \times b} = \frac{6F\left(\frac{d^2}{4} - y^2\right)}{bd^3}$ *

SHEAR STRESS DISTRIBUTION OVER SOLID CIRCULAR SECTION:

Show that the ratio of max. shear stress to mean shear stress in a solid circular section is equal to 4/3 when it is sub.t a transverse shearing force.



Let section AA be at a distance $\bar{y}$ from the N.A.

Consider a strip of thickness dy at a distance y from NA & at an angular distance θ from the N.A.

width of strip $= 2x = 2\left(\frac{D}{2}\cos\theta\right) = D\cos\theta$

and its distance $y = \frac{D}{2}\sin\theta \Rightarrow \frac{dy}{d\theta} = \frac{D}{2}\cos\theta$

$\therefore dy = \frac{D}{2}\cos\theta\, d\theta$, Also $I = \frac{\pi}{64}D^4$.

Moment of area above the level AA about N.A.

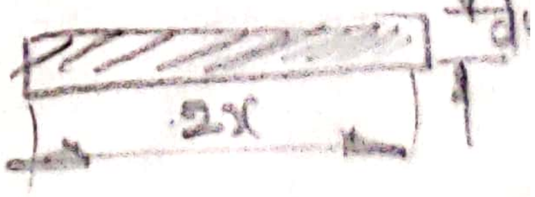
$$a\bar{y} = \int_{\theta_1}^{\pi/2} (D\cos\theta\, dy)\frac{D}{2}\sin\theta.$$

$$= \int_{\theta_1}^{\pi/2} D\cos\theta \left(\frac{D}{2}\cos\theta\, d\theta\right)\frac{D}{2}\sin\theta$$

$$= \frac{D^3}{4}\int_{\theta_1}^{\pi/2} \cos^2\theta \sin\theta\, d\theta.$$

[$\because a$ = area of strip $= 2x \times dy$
$a = D\cos\theta\, dy$.
$\bar{y} = \frac{D}{2}\sin\theta$.]

[$\because dy = \frac{D}{2}\cos\theta\, d\theta$]

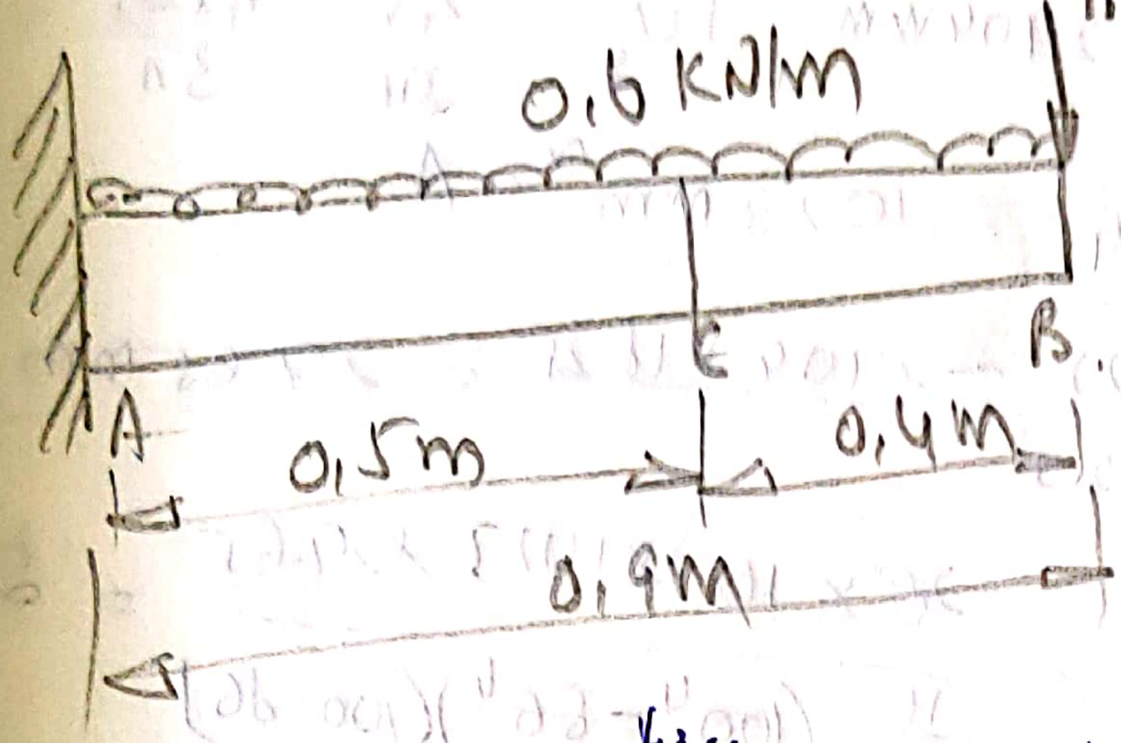


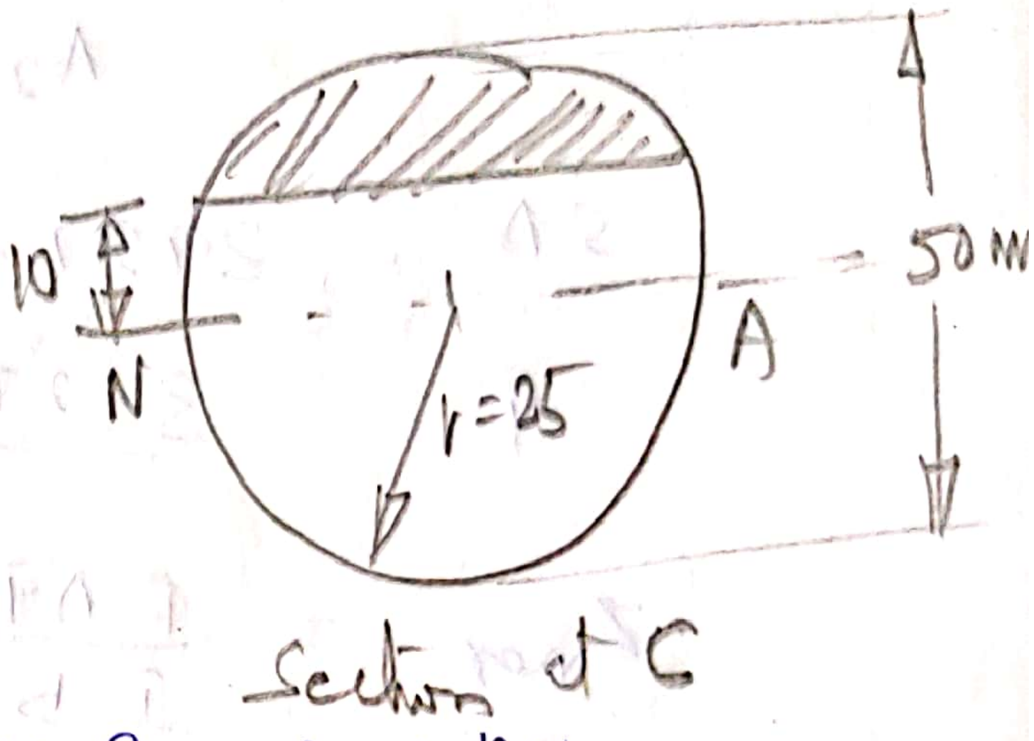
(4). A beam of circular solid section of 50mm dia is used as a cantilever of span 0.9m. It carries a concentrated load of 1.5KN at the free end. If the UDL on the beam is 0.6 KN/m. determine

(i) Max avg shear stress anywhere in the beam

(ii) max shear stress anywhere

(iii) shear stress in a fibre at 10mm from N.A on a section located at 0.4m from free end





Section at C

(1) Max shear force at A, $1.5 + 0.6 \times 0.9 = 2.04$ KN.

Max avg shear stress. $\tau_{avg} = \frac{F}{A} = \frac{2.04 \times 10^3}{\pi/4 \times 50^2} = 1.038$ N/mm^2

(2) $\tau_{max} = 1.33\, \tau_{avg} = 1.33 \times 1.038 = 1.385$ N/mm^2

(3) At Section C, 0.4m from free end.

$F = 1.5 + 0.6 \times 0.4 = 1.74$ KN.

$y = 10$ mm & $R = 25$ mm. $\Rightarrow d = 50$ mm.

$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (50)^4 = 306796$ mm^4

$\tau = \frac{F}{3I} R^2 - 4^2 = \frac{1.74 \times 10^3}{3 \times 306796} \cdot (25^2 - 10^2) = 0.994$ N

(5) A hollow circular section of O.D = 100 mm & I.D = 86 mm & S.T a shear of 28 KN. Determine the max shear stress & their ratio.

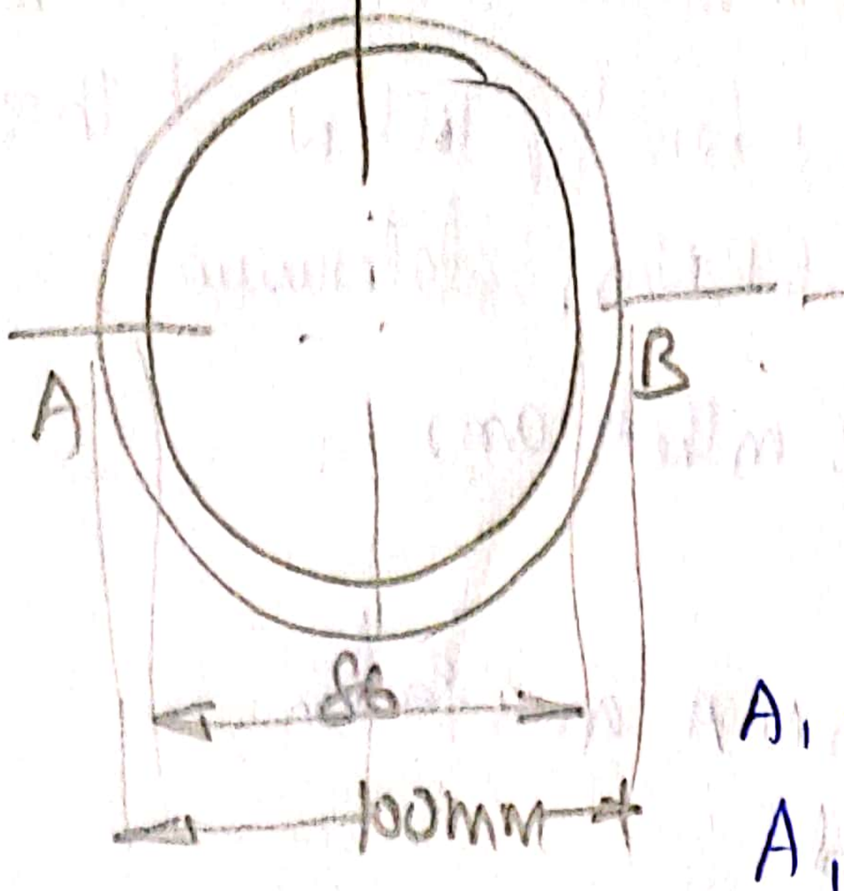
Max shear stress occurs at N.A AB

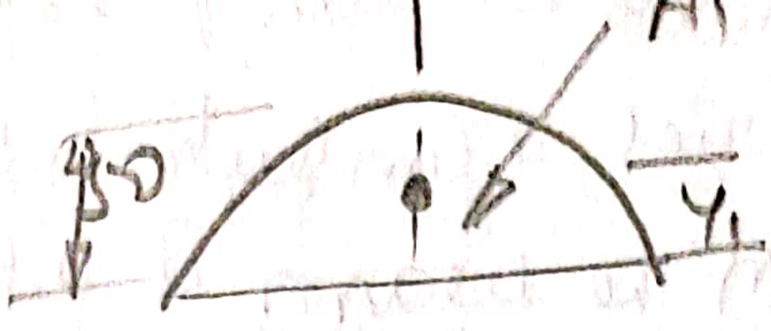
Partial area of section above N.A

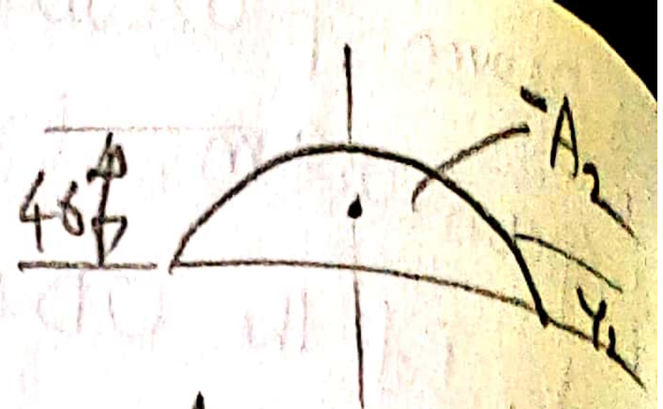
$A = \frac{\pi}{8} (100^2 - 86^2)$

$= 1022$ mm^2

[∵ Semi Circular area







$A_1 = 3927\ mm^2$

$\bar{y}_1 = 21.22$

$A_2 = -2904\ mm^2$

$\bar{y}_2 = 16.25$

$A_1 = \frac{\pi R^2}{2} = \frac{\pi (50)^2}{2}$

$A_1 = 3927\ mm^2$

$A_2 = -2904\ mm^2$

$\bar{y}_1 = \frac{4r}{3\pi} = \frac{4 \times 50}{3\pi} = 21.$

$\bar{y}_2 = \frac{4r}{3\pi} = \frac{4 \times 43}{3\pi} = 16.$

$\Sigma A = 3927 - 2904 = 1023\ mm^2 = A$

$\bar{y} = \frac{3927 \times 21.22 - 2904 \times 16.25}{1023} = 29.65\ mm.$

$Z_{max} = \frac{F A \bar{y}}{I b} = \frac{28 \times 10^3 \times 1023 \times 29.65}{\frac{\pi}{64}(100^4 - 86^4)(100 - 86)} = 27.$

$Z_{avg} = \frac{F}{Area} = \frac{28 \times 10^3}{2046.} = 13.68\ N/mm^2$ [$2 \times 1023 = 20$

$\frac{Z_{max}}{Z_{avg}} = \frac{27 - 28.}{13.68} \simeq 2 \quad \therefore Z_{max} \simeq 2 Z_{avg}$

(1) A rectangular beam section 150 mm wide x 300 mm depth is subjected to shear force 20 kN. Determine the avg shear stress & max. shear stress developed in the section.

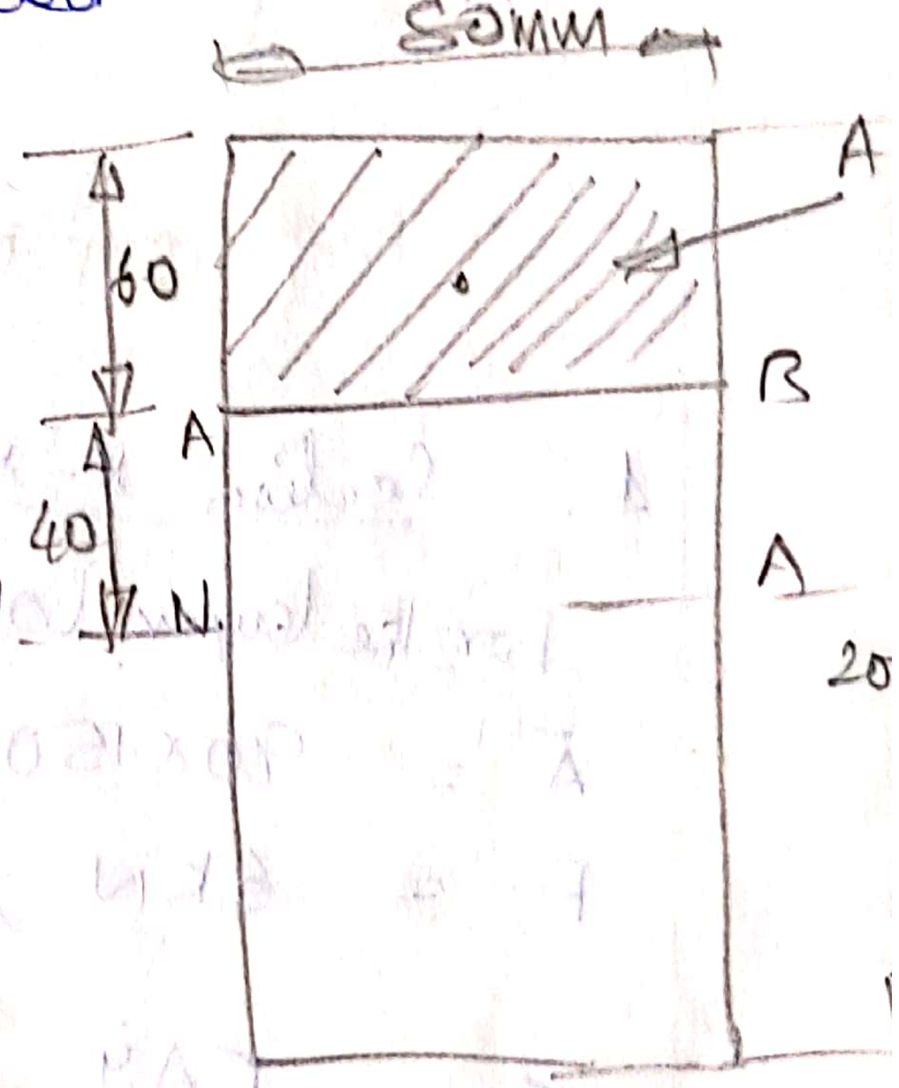
$F = 20kN$, $b = 150mm$, $d = 300mm$.

$\tau_{avg} = \frac{F}{bd} = \frac{20 \times 10^3}{150 \times 300} = 0.444 N/mm^2$, $\tau_{max} = 1.5 \tau_{avg} = 0.666$ @ N.A

(2) The avg shear stress at a section of a S.S.B having a dp of 50 mm x 200 mm depth is 0.5 N/mm². Determine

1) Shear force in the section
2) Max shear stress in the section
3) Shear stress at a pt one section located at 40 mm above N.A.

b = 50 mm
d = 200 mm.



1) $\tau_{avg} = \frac{F}{bd} = 0.5 N/mm^2$

$\therefore$ Shear force $F = 0.5 \times 50 \times 200 \times 10^{-3}$
$= 5 kN$.

2) $\tau_{max} = 1.5 \tau_{avg}$.
$= 1.5 \times 0.5 = 0.75 N/mm^2$

3) The fibre AB @ 60 mm above N.A.

$A = 60 \times 50 = 4500 mm^2$

$\bar{y} = 40 + \frac{60}{2} = 70 mm$.

$I_{NA} = \frac{1}{12} \times 50 \times 200^3 = 5.33 \times 10^7 mm^4$

$\tau_{AB} = \frac{F A \bar{y}}{I b} = \frac{5 \times 10^3 \times 4500 \times 70}{5.33 \times 10^7 \times 50} = 0.63 N/mm^2$.

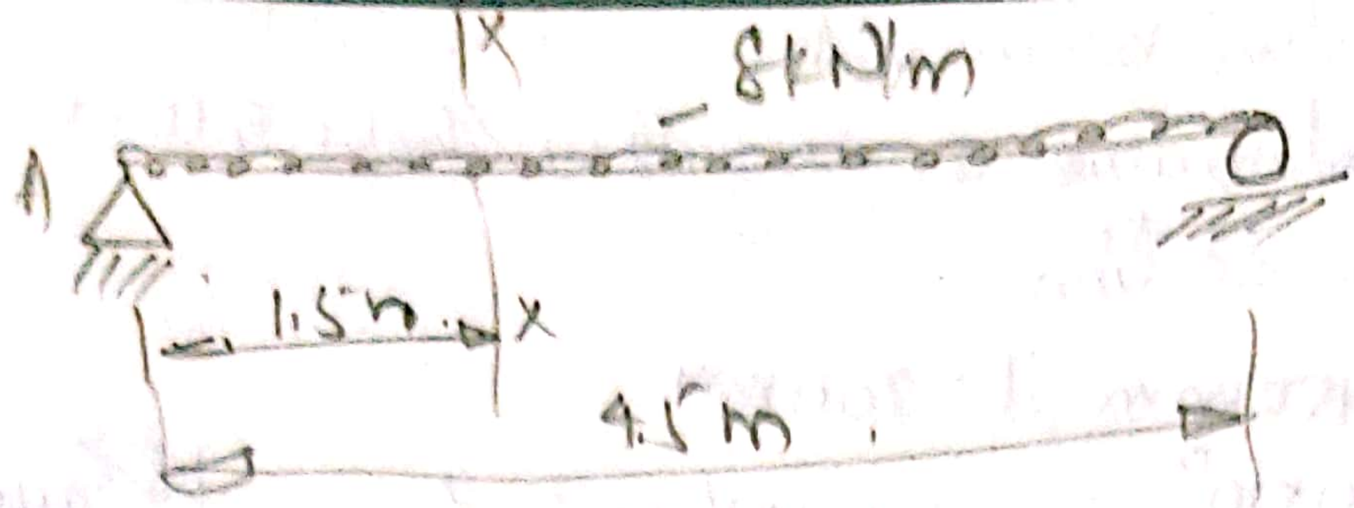
(3). A simply. S.B 150 mm wide x 300 mm deep is simply supported over a span of 4.5 m. It is loaded with a uniform load of 5 kN/m

Compute:

1) Shear stress developed on a layer 60 mm above the N.A of section located at 1.5 m from the left support
2) Max. shear stress on the above section.
3) Max shear stress anywhere in the beam.

Ans (1) Shear force on the section at 1.5 m from left support is 6 kN. The max shear stress anywhere in the beam is ~~of~~ the max shear stress ~~anywhere~~ in the beam.

$I = \frac{1}{12} 150 \times 300^3 = 3.375 \times 10^8 mm^4$.

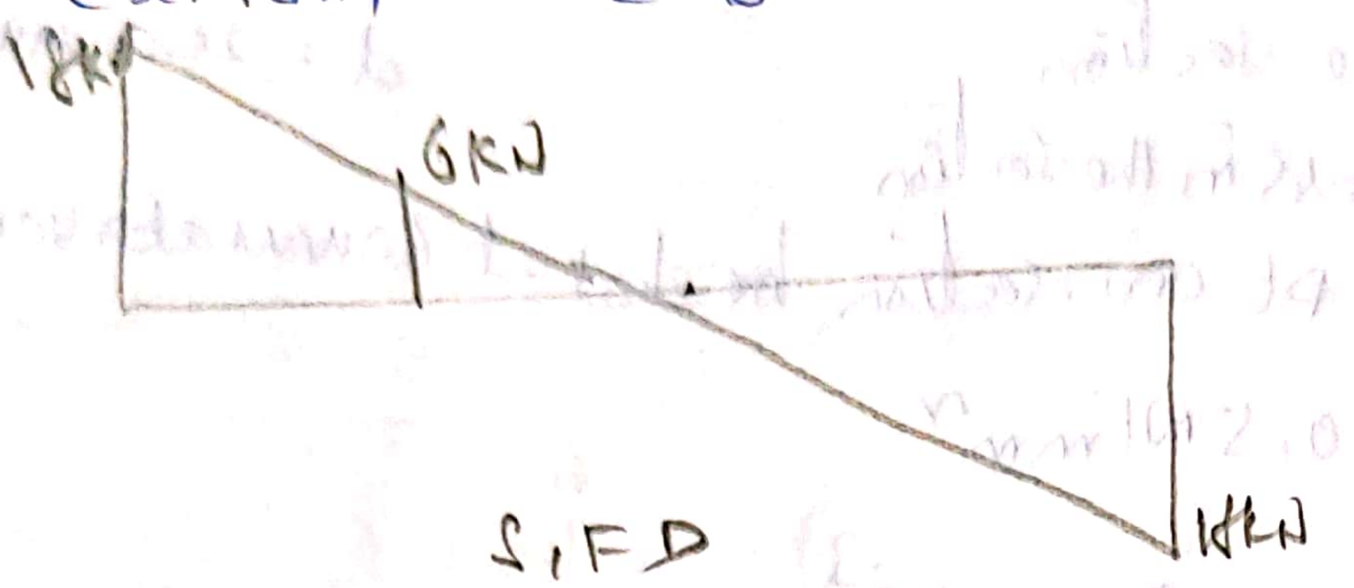


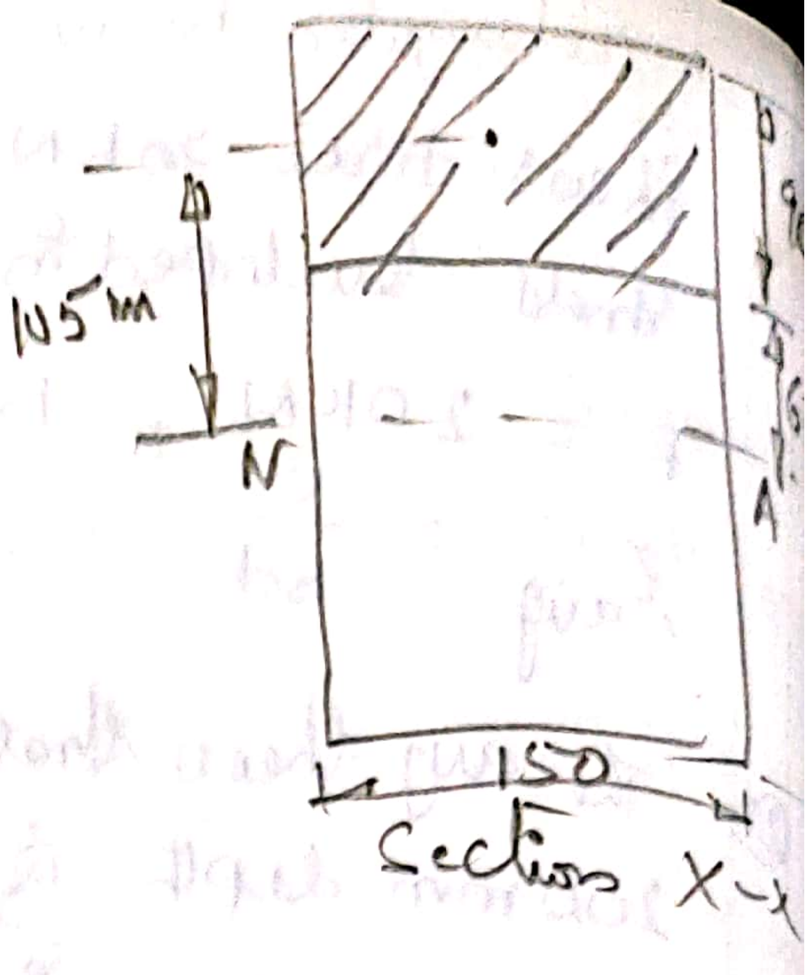
Total load = $8 \times 4.5 = 36$ kN.

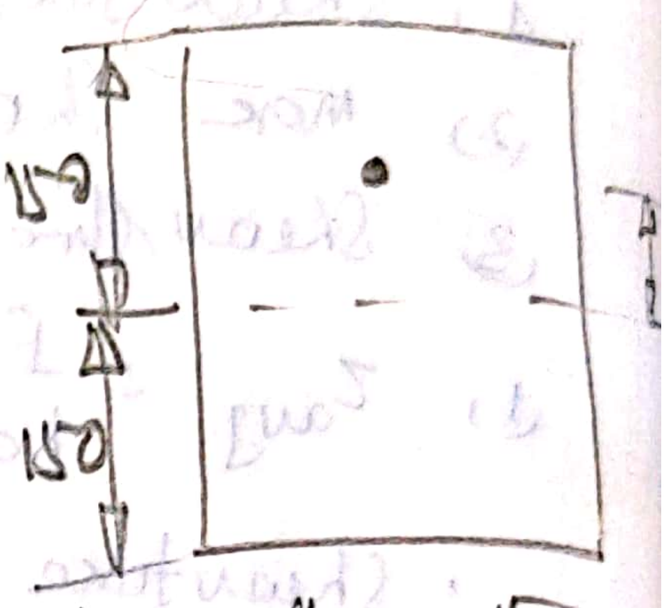
$R_A = 18$, $R_B = 18$

S.F @ x-x $F_x = R_A - 6 \times 1.5$

@ x > 1.5m $= 18 - 12 = 6$ kN.







1. Section x-x. @60mm

For the layer located above N.A.

$\bar{A} = 90 \times 150 = 13500\ mm^2$, $\bar{y} = 60 + 45 = 97.5\ mm$

$F = 6$ kN, $b = 150$ mm. = width of section at the point (layer being considered)

$\tau = \frac{FA\bar{y}}{Ib} = \frac{6 \times 10^3 \times 13500 \times 105}{3.375 \times 10^6 \times 150} = 0.168\ N/mm^2$.

2) Max Shear Stress on Section x-x: will be on N.A.

$\therefore A = 150 \times 150 = 22500\ mm^2$, $\bar{y} = 75$ mm.

$F = 6$ kN, $b = 150$ mm

$\tau = \frac{F.A\bar{y}}{Ib} = \frac{6 \times 10^3 \times 22500 \times 75}{3.375 \times 10^6 \times 150} = 0.2\ N/mm^2$

3) Max. Shear Stress any where in the beam: properties are same as 2

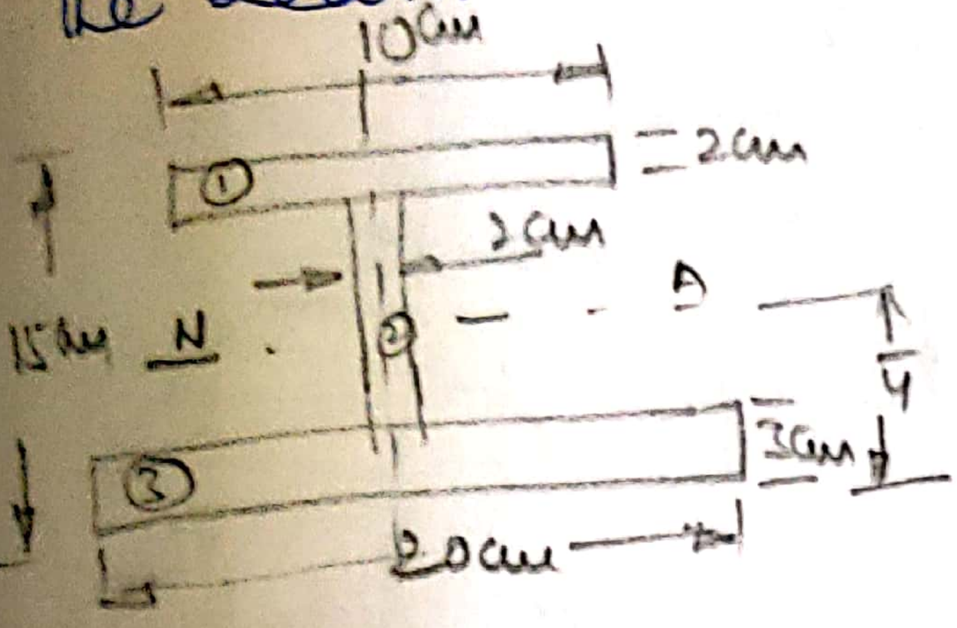
$S.F = 18$ kN, $\tau = \frac{FA\bar{y}}{Ib} = \frac{18 \times 10^3 \times 22500 \times 75}{3.375 \times 10^6 \times 150} = 0.6\ N/mm^2$

Also for rectangular section $Z_{max} = 1.5 Z_{avg}$ (F/A)

$\therefore$ For section x-x $Z_{max} = 1.5 Z_{avg} = 1.5 \times \frac{6 \times 10^3}{150 \times 300} = 0.2$

" " @ support " " " $= 1.5 \times \frac{18 \times 10^3}{150 \times 300} = 0.6$

The cross section of a Cast-iron beam is shown. This beam is simply supported at the ends and carries a uniformly distributed load of 20 kN/m. If the span of the beam is 3m. determine max tensile & compressive stresses in the beam.



$\bar{y}$ from bottom edge

$$\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{a_1\bar{y}_1 + a_2\bar{y}_2 + a_3\bar{y}_3}{a_1 + a_2 + a_3}$$

$$= \frac{(10\times2)14 + (2\times10)8 + (3\times20)(1.5)}{(10\times2) + (2\times10) + (3\times20)}$$

$$\bar{y} = \frac{530}{100} = 5.3$$

(or) Taking moments about the bottom edge of beam.

$$(10\times2 + 2\times10 + 20\times3)\bar{y} = (10\times2)14 + (2\times10)8 + (20\times3)1.5$$

$$\therefore \bar{y} = \frac{530}{100} = 5.3 \text{ cm}$$ multiplied

[$\because$ Each area multiplied by $\bar{y}$ = Each area about bottom edge

Total Sum

$$\therefore I = I_1 + I_2 + I_3$$

$$= \frac{b_1 d_1^3}{12} + a_1 d_1^2 + \frac{b_2 d_2^3}{12} + a_2 d_2^2 + \frac{b_3 d_3^3}{12} + a_3 d_3^2$$

[d_1, d_2, d_3 = difference b/t Each area C.G to bottom edge from bottom edge

$$= \frac{10\times2^3}{12} + (10\times2)(14-5.3)^2 + \frac{2\times10^3}{12} + (2\times10)(8-5.3)^2$$

$$+ \frac{3\times20^3}{12} + (1.5-5.3)^2$$

$$= 27646.33 \text{ cm}^4 = 27646.33\times10^{-8} \text{ m}^4.$$

for simply supported $M = \frac{wL^2}{8} = \frac{20(3)^2}{8} = 22.5 \text{ kN-m}.$

$$\frac{M}{I} = \frac{\sigma_c}{y}.$$

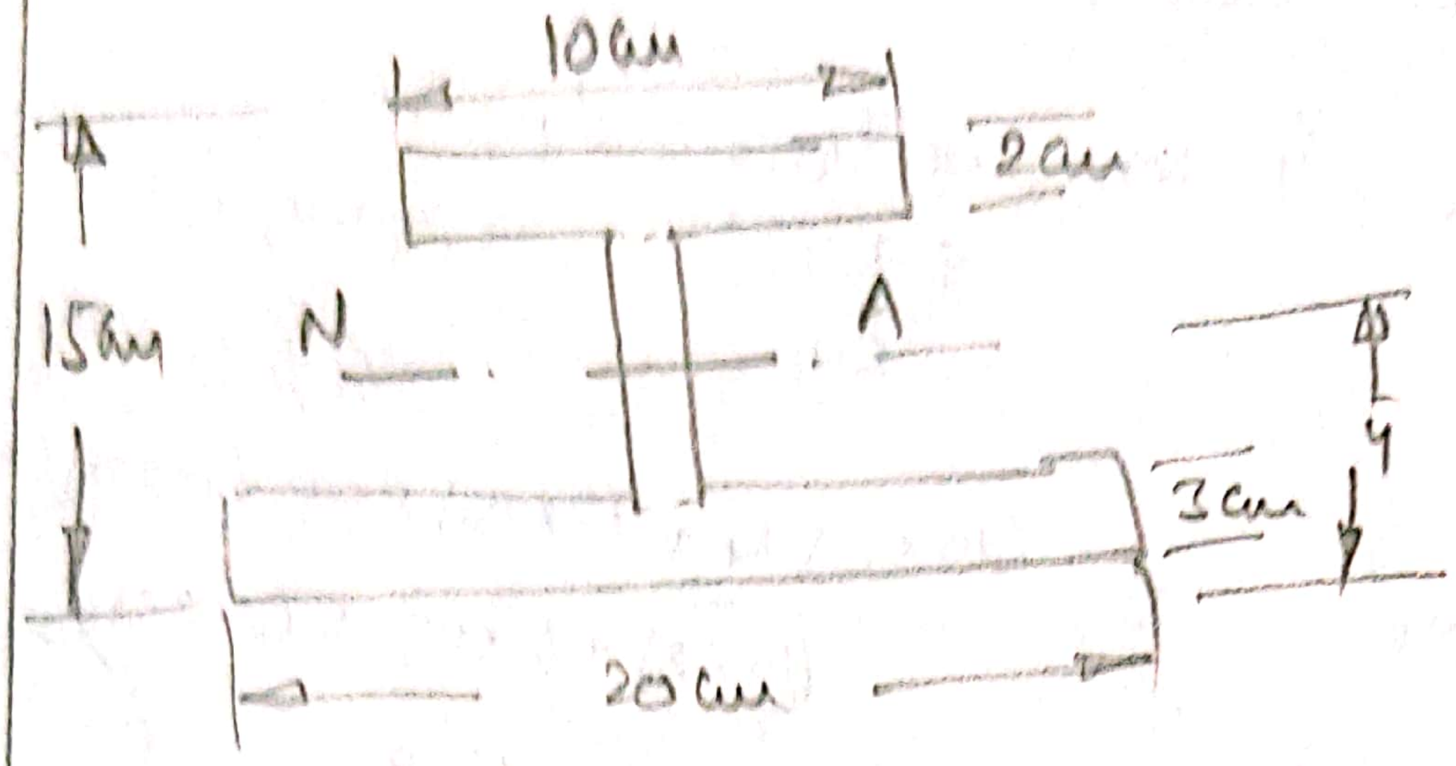
$$\therefore \sigma_c = \frac{M}{I}y = \frac{22.5\times10^3}{27646.33\times10^{-8}}(15-5.3)10^{-2}.$$

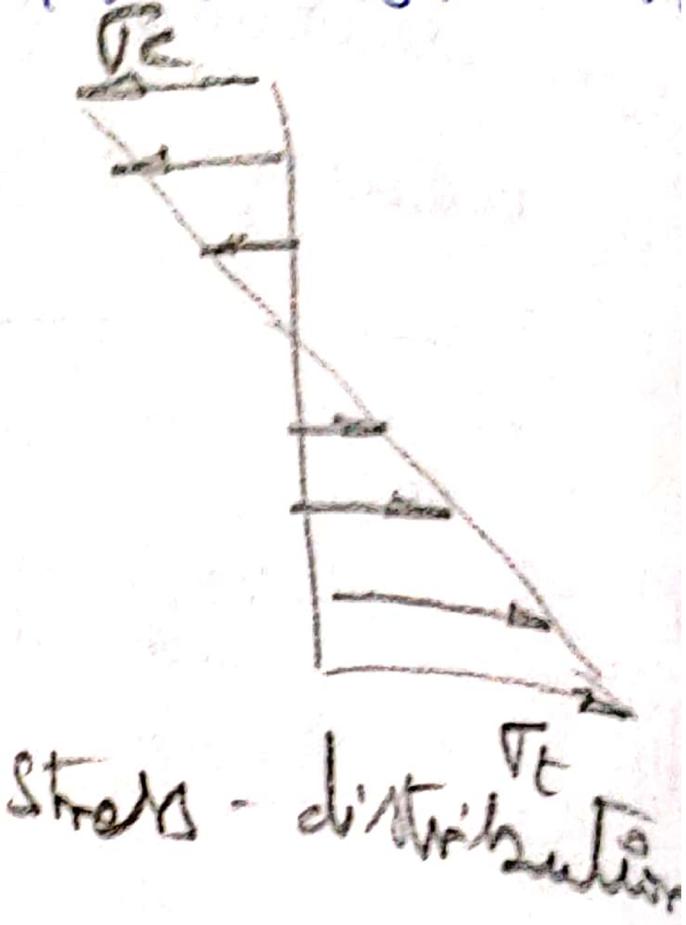
At top distance (15 - $\bar{y}$)

: Max Compressive bending stress σ_c = 79.65

Max Tensile bending stress

$$\sigma_t = \frac{M}{I}\bar{y} = \frac{22.5\times10^3\times5.3\times10^{-2}}{2744.33\times10^{-8}} = 43.$$

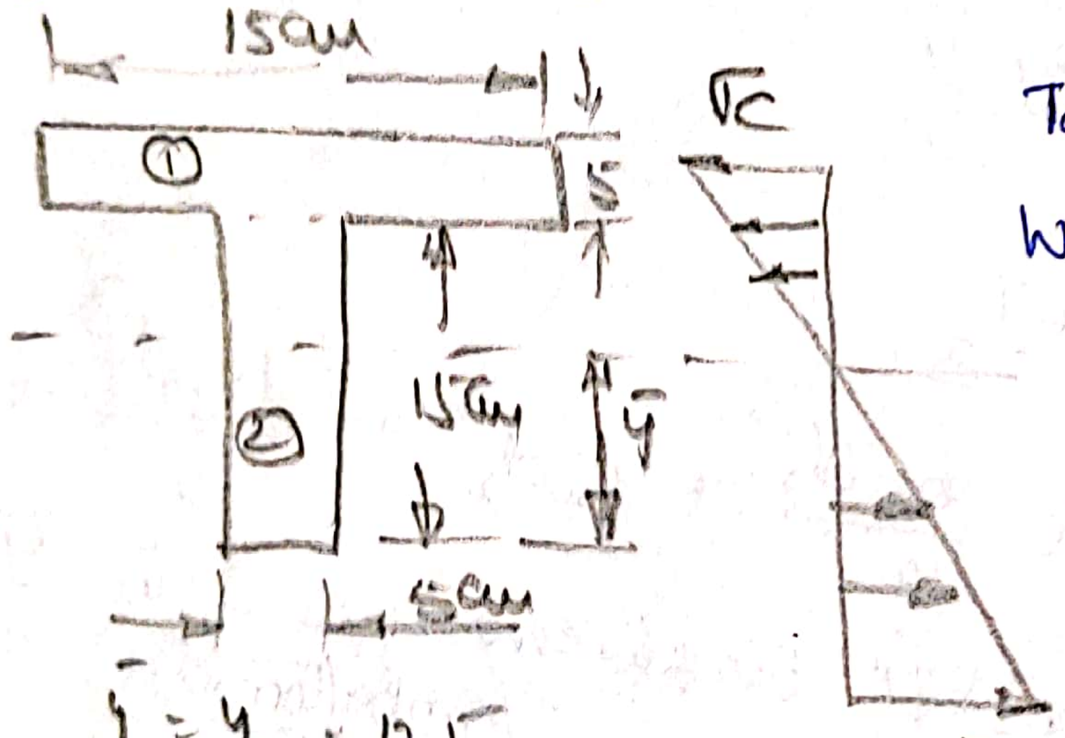




Stress - distribution

2283

7.3 Two wooden planks 5cm x 15cm each are connected together to form a C/S of a beam as shown. If a bending moment of 3400 N-m is applied around the horizontal find the stress as at the extreme fibres of the C/S.



$\bar{y} = y_t = 12.5$

$\bar{y} = y_c = 20 - 12.5 = 7.5$

Taking moment about bottom we have

A_1 A_2 $A\bar{y}_1 + A$

$(15\times5 + 15\times5)\bar{y} = 15\times5\times17.5 + 15$

$150\bar{y} = 1875$

$\bar{y} = 12.5$ cm.

$\left[\because \bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A}\right]$

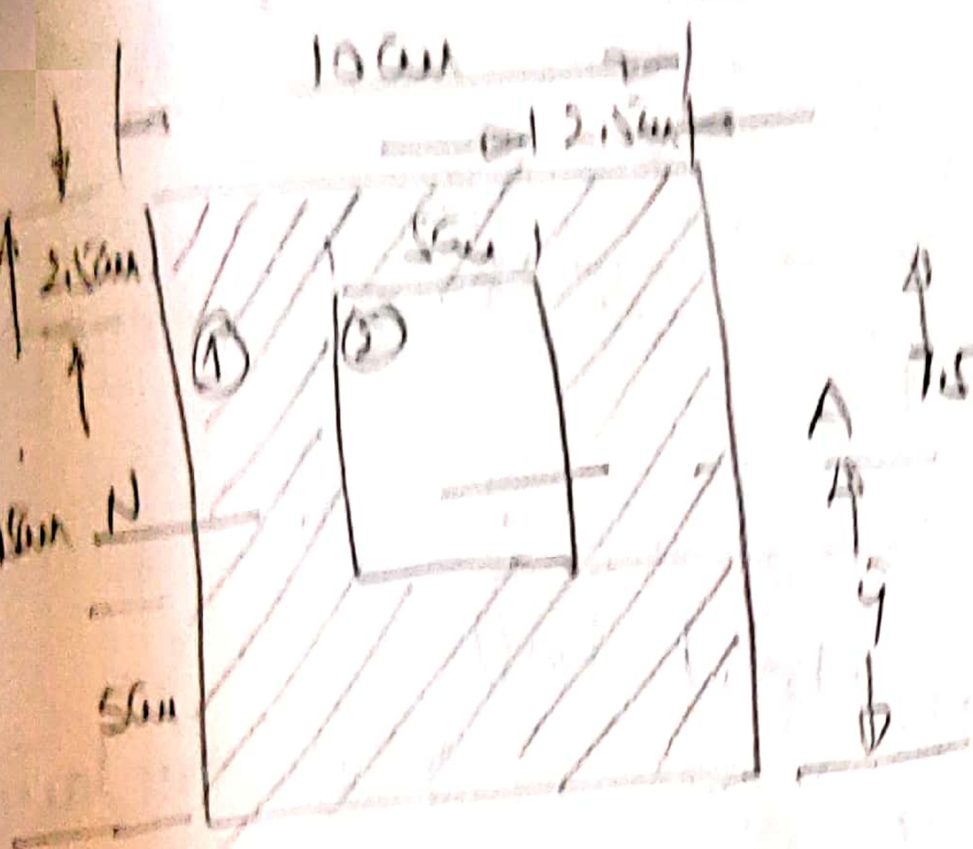
$$I = \frac{15\times5^3}{12} + (15\times5)(17.5-12.5)^2 + \frac{5\times15^3}{12} + 15\times15](7$$

$$= 5312.5\ cm^4 = 5312.5\times10^{-8}\ m^4.$$

$$\sigma_t = \frac{M}{I}\bar{y}_t = \frac{3400\times12.5\times10^{-2}}{5312.5\times10^{-8}} = 8\ MPa.$$

$$\sigma_c = \frac{M}{I}\bar{y}_c = \frac{3400\times7.5\times10^{-2}}{5312.5\times10^{-8}} = 4.8\ MPa.$$

The cg of a cast iron machine element used as a beam, the beam having bending moments about the horizontal neutral axis, the permissible stresses in tension & compression are to be 22 & 84 MPa respectively. Calculate the moment of resistance of the section about the horizontal neutral axis for both +ve & -ve bending moments.



$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

$$= \frac{(15\times10)\,7.5 - (7.5\times5)\left[5 + \frac{7.5}{2}\right]}{(15\times10) - (7.5\times5)}$$

$$\bar{y} = 7.04 \text{ cm}.$$

$$\therefore I = \frac{10\times15^3}{12} + (10\times15)(7.5-7.04)^2 - \frac{5(7.5)^3}{12} - (5\times7.5)(5.75-7.04)^2$$

$$= 2812.5 + 26.46 - 115.74 - 104.56 \quad I = I_1 - I_2$$

$$= 2558.6 \text{ cm}^4 = 2558.6\times10^{-8}\ m^4.$$

For +ve moment, Stress at top is Compressive & at the bottom is tensile.

$\therefore \sigma_t = 22 \text{ MPa}$

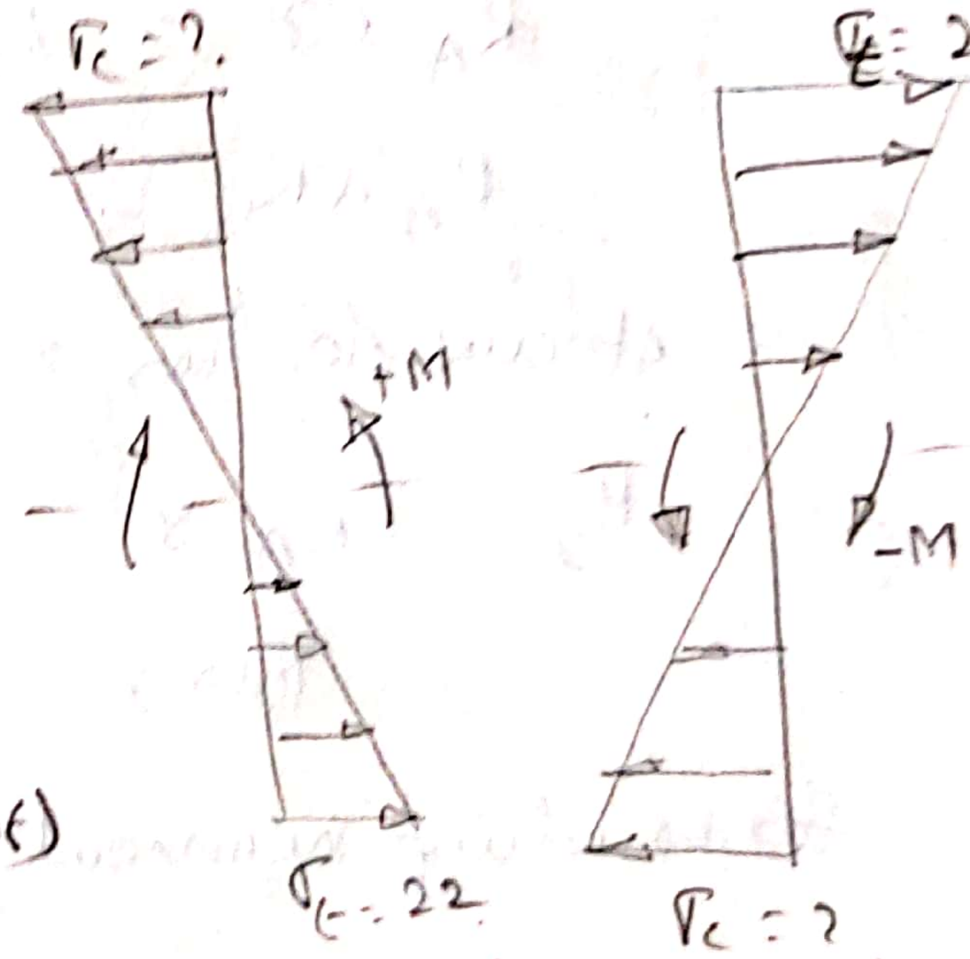
$$\sigma_c = \frac{22\times7.92}{7.08}$$

σ from bottom

7.08 - 22 MPa

7.92 - ?

↑ at top (15 - 7.08)



Moment of Resistance $M = \frac{\sigma_t}{y} \times I = \frac{22\times10^6 \times 2558.6\times10^{-8}}{7.08\times10^{-2}} = 7950 \text{ N-m}$

For -ve B.M, Stress at top will be tensile.

$\therefore$ Stress at the bottom, $\sigma_c = \frac{7.08}{7.92} \times 22 = 19.67 \text{ MPa}$

7.92 - 22

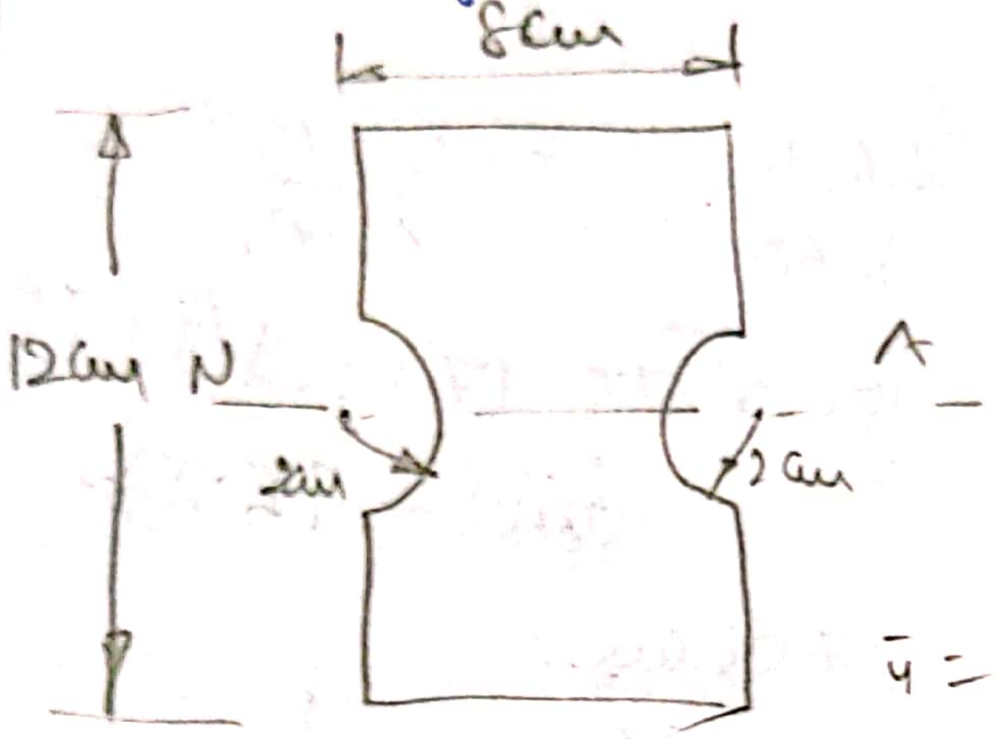
7.08 - ?

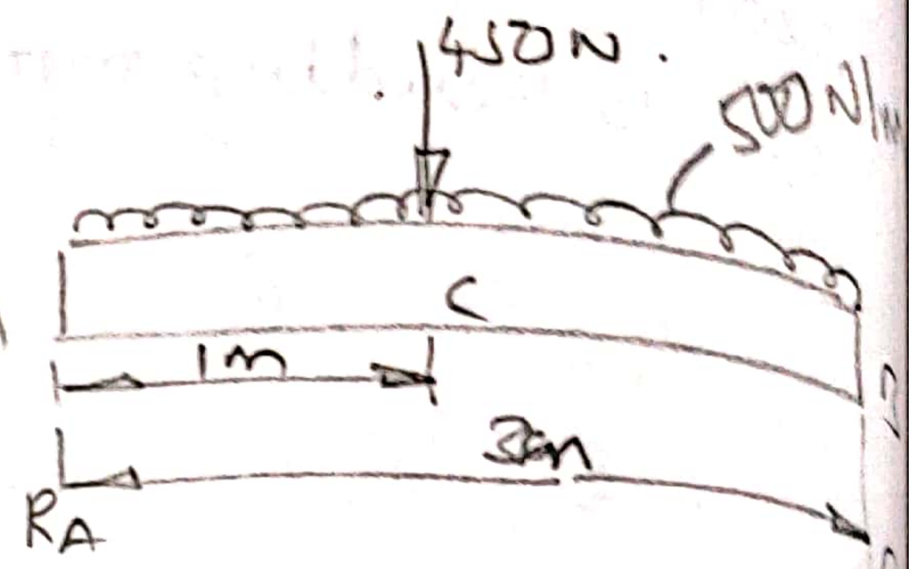
$\frac{7.08\times22}{7.92}$

$$M = \frac{22\times10^6 \times 2558.6\times10^{-8}}{7.92\times10^{-2}} = 7107.2 \text{ N-m}$$

Scanned with Oken Scanner

A wooden beam is [illegible] with a circular groove of 2cm radius planned out in the middle of each side. Calculate the max stress in the section when the beam supported on a span of 3m, loaded with a concentrated load of 450N at a distance of 1m from the one end and a U.D.L of 500N/m run over the whole span.





$$\bar{y} = \frac{12}{2} = 6cm = 6\times10^{-2}m$$

Since the beam bending about both axes.

$$I = \frac{8\times12^3}{12} - \frac{\pi}{64}4^4$$

[I for rectangle, I for circle]

$$= 1139.43cm^4 = 1139.43\times10^{-8}m^4.$$

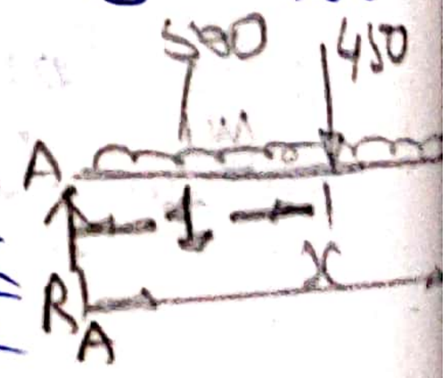
Two semi Circles

Now for the beam loaded $\Sigma m_B = 0$.

$$R_A\times3 = 450\times2 + 500\times3\times\frac{3}{2} \Rightarrow R_A = \frac{3150}{3}$$

$$R_A + R_B = 450 + 500\times3 = 1950 \Rightarrow R_B = 900N$$

∴ At any section x from A @ distance x:



$$M_x = R_A x - 450(x-1) - 500x\times\frac{x}{2}$$

$$= 1050x - 450x + 450 - 250x^2 = 600x + 450 -$$

For bending moment to be max $\frac{dM_x}{dx} = 0 \Rightarrow 600 - 500x$

$x = 1.2$

$$M_{max} = 600\times1.2 + 450 - 250(1.2)^2 = 720 + 450 - 360 = 810$$

Apply B.M Eq $\frac{M}{I} = \frac{\sigma_b}{y}$

∴ Max bending stress $\sigma_b = \frac{M_{max}\times y}{I}$.

$$= \frac{810\times6\times10^{-2}}{1139.43\times10^{-8}} = 4.265MN/m^2$$

A rolled steel Joist 450 mm x 200 mm is freely supported on a span of 4.5 m. The flanges are strengthened by two 300mm x 20mm plates, one riveted to each flange. The second moment of area of Joist is $35,060 cm^4$. Calculate

(a) the greatest central load the beams will carry if the stress in the plates is not to exceed 120 MPa.

(b) the min length of the 300mm plates required to restrict the maximum stress in the flanges of Joist to 120 MPa.

Sol: Moment of inertia about N.A

I = Moment of Joist + Moment of 2 beams

$I = 35060 + 2\left[\frac{30 \times 2^3}{12} + 30 \times 2 (23.5)^2\right]$

$= 35060 + 2(20 + 33,130)$

$= 101,370 cm^4 = 101370 \times 10^{-8} m^4$.

$\therefore Z = \frac{I}{y} = \frac{101370}{24.5} = 4138 cm^3 = 4138 \times 10^{-6} m^3$.

(a) $M = \sigma \times Z = 120 \times 10^6 \times 4138 \times 10^{-6} = 496.56$ kN-m

[$\because \sigma = 120$ MPa $= 120 \times 10^6$ Pa]

Max bending moment $M = \frac{WL}{4} = \frac{W \times 4.5}{4}$ [$L = 4.5$ m
$\because$ Simply suppo

$W = \frac{4 \times 496.56}{4.5} = 441.4$ kN.

(b) Let the plates be installed at a distance of x metres from each support

Moment at this section $M = \frac{W}{2} \times x = \frac{441.4 \times 10^3}{2} \times 2$.

$= 220.7 \times$ kN-m

$I = 35060 cm^4$.

$Z = \frac{I}{y} = \frac{35060}{22.5} = 1558 cm^3$ [$\bar{y}$ for $\square^{lr} = \frac{45}{2} = 22.5$

$= 1558 \times 10^{-6} m^3$.

$\therefore M = \sigma \times Z$

$220.7 \times 10^3 x = 120 \times 10^6 \times 155.8 \times 10^{-6}$ m3

$x = 0.847$ m.

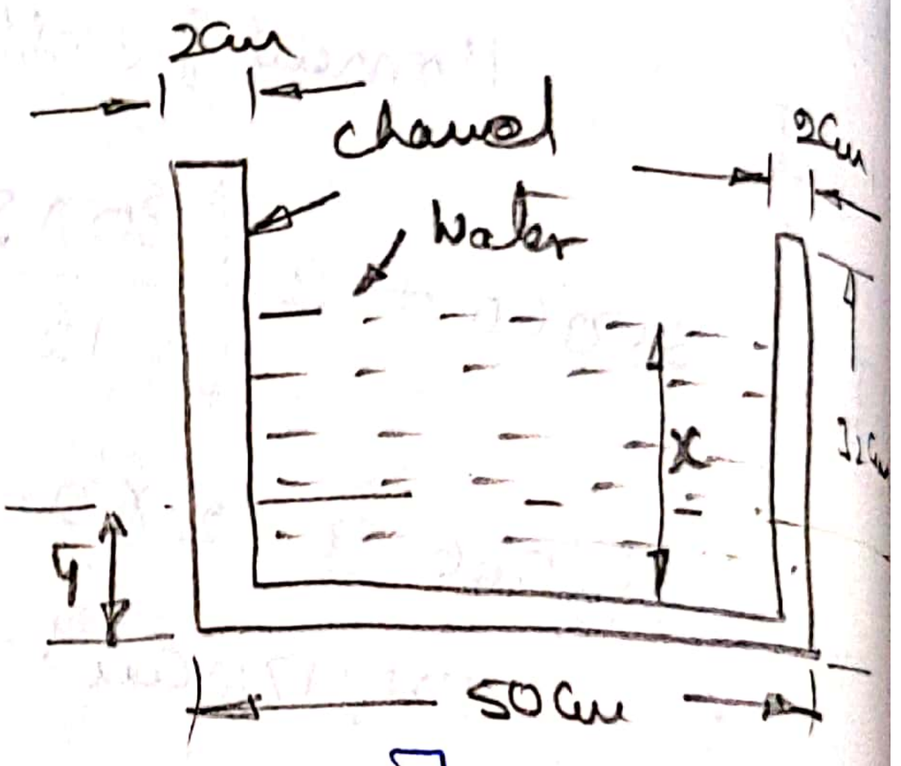
$\therefore$ Minimum length of plates $= 4.5 - 0.847 \times 2 = 2.$

P289

7.10. A cast iron channel, whose section is shown in fig, carries water. It is supported at two points 10 m apart. Weight of cast iron weight 7000 kg/m3. Find the depth of water in the channel if the tensile and compressive bending stresses are not to exceed by 15 & 45 MPa

Sol. $\bar{y} = \frac{50 \times 2 \times 1 + 2 \times 30 \times 2 \times 17}{50 \times 2 + 2 \times 30 \times 2}$ (15 + 2)

$= 9.73$ cm.



M.O.I about N.A

$I = \frac{50 \times 2^3}{12} + 50 \times 2[9.73 - 1]^2$

$+ 2\left[\frac{2 \times 30^3}{12} + 2 \times 30(17 - 9.73)^2\right]$

$= 7650 + 15,250 = 23000$ cm^4 $= 23 \times 10^{-5}$ m^4.

Moment of resistance in Compression $\left[\because \frac{M}{I} = \frac{\sigma_b}{y}\right]$

$M = \frac{45 \times 10^6 \times 23 \times 10^{-5}}{(32 - 9.73) \times 10^{-2}} = 46.5$ kN-m

$\sigma_b = 45$M
$= 45 \times$
$y_c = \bar{y} = (32 - 9.7$

(ii) M.O.R in tension $M = \frac{15 \times 10^6 \times 23 \times 10^{-5}}{9.73 \times 10^{-2}}$

$= 35.46$ kN-m

$\left[\because \sigma_b = \sigma_t\right.$
$\left. y = y_t =\right.$

$\therefore$ Safe bending moment $= 35.46$ kN-m. $\therefore \sigma_t = 15$M

& $\sigma_c = 15 \times \frac{22.27}{9.73} = 34.33$ MPa.

9.73 — 15M
22.27 — σ_c

let w = Total weight of channel & water / m length in KN.

$\therefore M_{max} = \frac{wl^2}{8} = \frac{w \times 10^2}{8} = 12.5w$ kN-m [∵ for simply supported]

$\therefore 12.5w = 35.46 \Rightarrow w = 2.836$ kN/m

*Wt of channel / m $= [50 \times 24 - 2 \times 30 \times 2] 10^{-4} \times 7000 \times 9.81$

$= 220 \times 10^{-4} \times 7000 \times 9.81$.

$= 1510.74$ N.

$\therefore$ Wt of water $= 2836.8 - 1510.74 = 1326.06$ N/m ---(1)

Let x = Depth of water in cm in the channel.

$\therefore$ Wt of Water/m $= 46 \times x \times 1000 \times 9.81 \times 10^{-4}$.

$= 45.126x$ N.

(50 - 4 = 46)

$\therefore$ From (1) $1326.06 = 45.126x$

$\Rightarrow x = \frac{1326.06}{45.126} = 29.38$ cm.

[NOTE: * Wt density : kg/m³.

* Wt/m $= \frac{kg}{m^3} \times m^2 = \frac{kg}{m} = \frac{kg \times 9.81 N}{m}$.

1. A steel beam having an I section is 4m long and is simply supported at the ends. If the safe stress in tension for the beam is 30 MPa, determine the permissible U.D.L acting on the whole span of beam.

Sol: Moment of Inertia about the NA

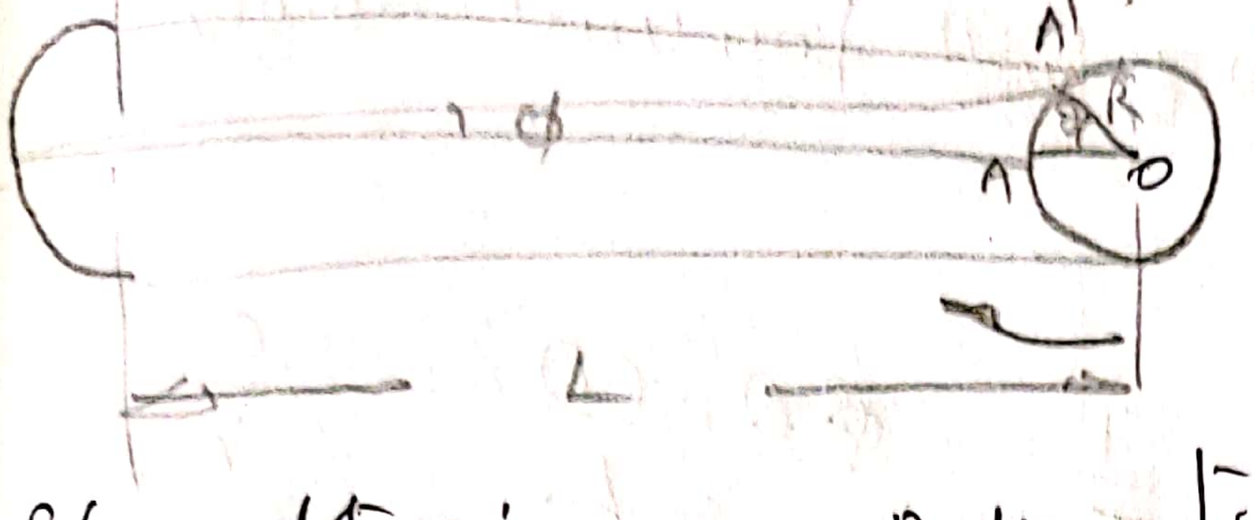
= Moment of Inertia of Rectangle 20cm x 30cm - 2[moment of inertia of Rectangle 9cm x 26cm).

TORSION

0.4
1

1 Introduction: The behaviour of a member under Torsion is very complex, when section is not axisymmetrical.

Torque Eq" for Axi Symmetrical Sections



Let $\angle ACA' = \phi$ in radians

$\angle AOA' = \theta$ "

τ = Shear Stress induced @ the surface

C = Modulus of rigidity

$\therefore$ Shear Strain $= \frac{\text{Deformation}}{\text{length}} = \frac{AA'}{L} = \tan\phi \simeq \phi$

But $AA' = R\theta \quad \therefore \phi = \frac{R\theta}{L}$ — (1)

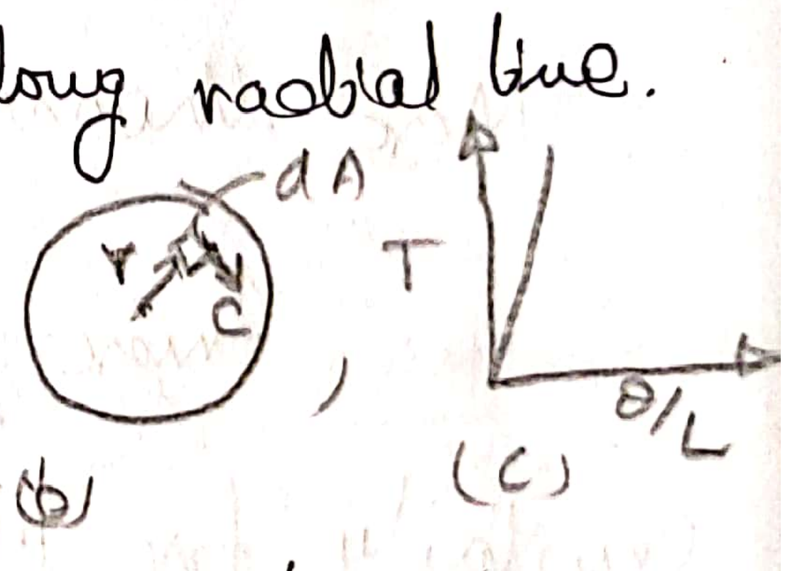
But $C = \frac{\tau}{\phi} = \frac{\text{Shear Stress}}{\text{Shear Strain}} \Rightarrow \phi = \frac{\tau}{C}$ — (2)

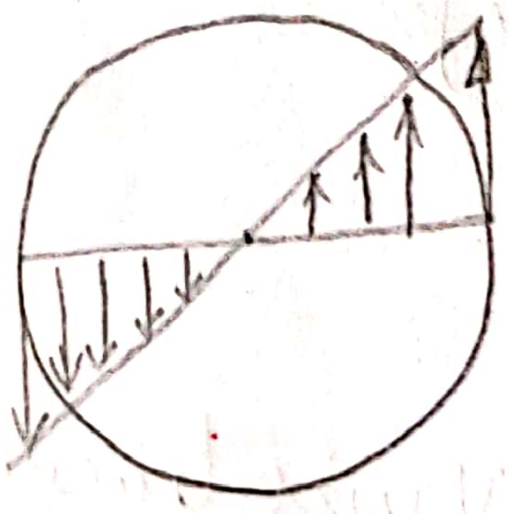
(1) = (2) $\frac{\tau}{C} = \frac{R\theta}{L} \Rightarrow \frac{\tau}{R} = \frac{C\theta}{L} = \frac{G\theta}{L}$.

Two imp Conclusions regarding Shear Stress distribution

) Shear Stress τ at any point in C/S is proportional to distance [from Longitudinal axis] of the point along radial line.

) Shear Stress acts ⊥r to radial line.





Shear Stress distribution

Relation b/t τ & Applied Torque

Applying equilibrium conditions, from fig (b)

The Internal Twisting moment = External applied Torque.

" $dT = \underbrace{\tau dA}_{\text{Force}} r$ [i.e. Torque = Force x Radius]

where τ is the Shear Stress on an elementary area dA @ distance r from the centre of Shaft.

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$\therefore T = \int_A \tau dA\, r = \int_A \left(\frac{G\theta}{L} r\right) r\, dA = \frac{G\theta}{L}\int_A r^2\, dA$

But $\int_A r^2 dA = J$ = Polar moment of Inertia $\left(\frac{G\theta}{L} = \frac{C\phi}{L}\right)$

$\therefore T = \frac{C\phi J}{L} = \frac{T}{J} = \frac{G\theta}{L} \Rightarrow \frac{T}{J} = \frac{G\theta}{L}$ (i)

where $\frac{\theta}{L}$ = Angle of twist per unit length.

But $\frac{G\theta}{L} = \frac{\tau}{R}$

$\therefore$ Torsion Equation $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$.

$\frac{T}{(\theta/L)} = CJ$ = Torsional Stiffness, Similar

EI = bending stiffness.

Since CJ is constant for a given shaft, a graph between Torque & Angle of twist per unit length will be st

Max. Shear stress for solid & Hollow Circular shaft:

For solid shaft, $J_{solid} = \frac{\pi}{32} D^4$.

$\therefore$ Max. shear stress on the surface $\frac{T}{J} = \frac{\tau_{max}}{R}$.

$\therefore \tau_{max} = \frac{T}{J} \cdot R = \frac{T}{\frac{\pi}{32} D^4} \cdot \frac{D}{2} = \frac{T \times 16}{\pi D^3}$.

Similarly for Hollow $J = \frac{\pi}{32}(D^4 - d^4)$

$$\tau_{max} = \frac{16\, T\, D}{\pi (D^4 - d^4)}$$

Shear stress is zero @ centre & varies linearly a

the radius.

Torsion Eqn: $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R} = \frac{\tau_r}{r}$.

where T = Twisting moment applied on shaft
= Resisting twisting moment in shaft
J = polar M.O.I of section of shaft

$\Rightarrow$ N

A shaft is subjected to a torque of 16000 N-m. If more permissible shear stress in the material of the shaft is 65 N/mm², find (i) Dia of solid shaft & (ii) the dimensions of a hollow circular shaft if its thickness is 10% of internal dia.

(i) $T = 16000\ N\text{-}m$, $\tau = 65\ N/mm^2$

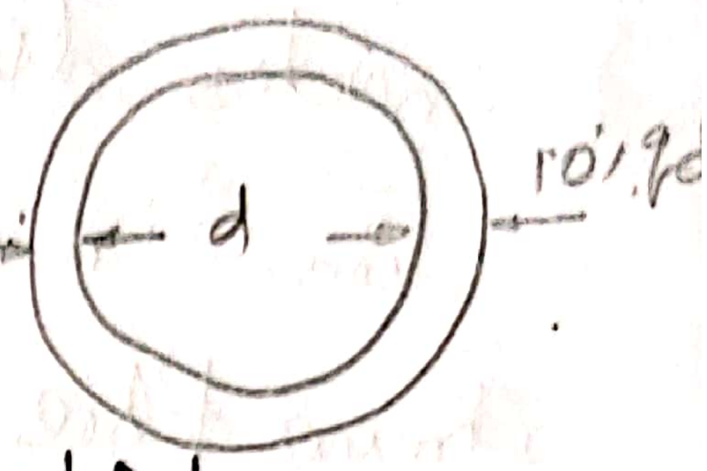
$$\tau_{max} = \frac{16T}{\pi D^3} \Rightarrow D^3 = \frac{16T}{\pi \tau_{max}} = \frac{16 \times 16000 \times 10^3}{\pi \times 65}$$

Dia of solid shaft $D = \sqrt[3]{1253651.21} = 108\ mm$.

Let inner dia $= d$.

outer dia $D = d + 0.1d + 0.1d$.

$= d + 0.2d = 1.2d$



$$\tau_{max} = \frac{16TD}{\pi(D^4 - d^4)} = \frac{16 \times 16000 \times 10^3 \times 1.2d}{\pi[(1.2d)^4 - d^4]}$$

$$= \frac{(1.2d)^4 - d^4}{d} = \frac{16 \times 16000 \times 10^3 \times 1.2}{\pi \times 65} =$$

$$= \frac{d^4[(1.2)^4 - 1]}{d} = 1504.84 =$$

polar moment of inertia $= \frac{\pi}{32}[D^4 - d^4] = \frac{\pi}{32}[(1.2d)^4 - d^4]$

$\therefore J = 0.105\, d^4$, & $\frac{T}{J} = \frac{\tau}{D/2} = \frac{\tau}{\frac{1.2d}{2}}$

$\therefore \frac{16 \times 16000 \times 10^3}{0.105\, d^4} = \frac{65}{0.6d} \Rightarrow d^3 = \frac{16000 \times 10^6 \times 0.6}{65 \times 0.105}$

$d = 106.7\ mm$, outer dia $= 128\ mm$.

Stamp page 1: $J = \frac{\pi R^4}{2}$ (for solid shaft) $= \frac{\pi}{2}(R_2^4 - R_1^4)$ for hollow shaft & outer radius R_2 & inner radius R_1

τ = max shear stress @ radius R in case of solid shaft @ radius R_2 in case of hollow shaft

G = Shear modulus of material

θ = Angular twist in shaft over length L of the shaft

τ = shear stress at any radius r of the shaft

Torsional Rigidity (GJ) (or) Stiffness:

Torsional stiffness can be defined as the torque required to produce unit rotation per unit length of the shaft.

We know $\frac{T}{J} = \frac{G\theta}{L} \Rightarrow GJ = \frac{TL}{\theta}$

For solid circular shaft, $GJ = G \cdot \frac{\pi}{32} d^4$.

Polar Section Modulus (Z_p): It is defined as the polar moment of inertia divided by the distance to extreme fibre of the section. In case of circular and hollow circular sections extreme fibre distance is either the radius (r) (or) diameter (d) radius (vc). Thus polar modulus $Z_p = \frac{J}{r}$ (or) J/r_e

From $\frac{T}{J} = \frac{\tau}{r} \Rightarrow$ Torque $= \frac{J\tau}{r} = Z_p \tau$.

$\therefore Z_{p\,solid\,shaft} = \frac{\pi d^3}{32}$, $Z_{p,hollow} = \frac{2\pi(d_e^4 - d_i^4)}{32 d_e}$.

Torsional Moment of Resistance: Max. torque that can be carried by the section without exceeding the max. permissible shear stress.

Torsional moment of resistance, $T_m = Z_p \times \tau$.

For solid, $T_{m,solid} = \frac{\pi d^3}{16} \cdot \tau$.

$T_{m;hollow} = \frac{\pi}{16} \frac{(d_e^4 - d_i^4)}{d_e} \tau$.

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Ex: A solid circular shaft has a diameter of 80mm. Find the maximum shear stress and the angle of twist in a length of 2m when the shaft is S.T torque of 10 KN-m. $G = 85$ GPa.

$\tau_{max} = \frac{TR}{J} = \frac{16T}{\pi D^3} = \frac{16 \times 10 \times 10^3 \times 10^3}{\pi (80)^3} = 99.5$ N/mm^2

Also $\frac{T}{J} = \frac{G\theta}{L}$, where θ is angle of twist over $L = 2m = 2000$ mm, $G = 8$

$\therefore \theta = \frac{TL}{GJ} = \frac{10 \times 10^6 \times 2000 \times 32}{8(200 \times \pi(80)^4}$ = 0.0

A hollow circular shaft has an external dia. ... external dia is three-fourths the external dia. If the shear ... a fibre inside is 36 MPa, due to torque T applied, find the ... que, the max shear stress, and the angle of twist per unit ... gth, $G = 85$ GPa.

$\tau = 36$ N/mm^2 (MPa)

$d_e = 120$ mm, $d_i = \frac{3}{4} \times 120 = 90$ mm, τ_{inside}.

$T = ?$, $\tau_{max} = ?$. Angle of Twist θ/unit length, $G = 85$ MPa

$J = \frac{\pi}{32}(d_e^4 - d_i^4) = \frac{\pi}{32}(120^4 - 90^4) = \frac{\pi}{32} \times 14,175 \times 10^4$.

$\tau = \frac{T}{J} r \Rightarrow T = \frac{\tau J}{r}$, here $r = 90$ mm [for τ]

$T = \frac{36}{90} \frac{\pi}{32} \times 14175 \times 10^4 = 5.567$ kN-m

The max shear stress τ_{max} will be @ surface fibres.

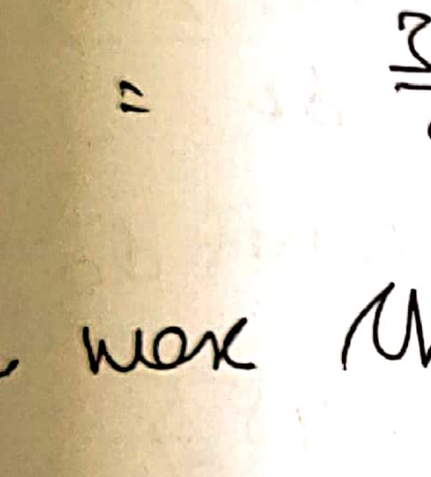
$\tau_{max} = \frac{T}{J} R$ [here $R = 120$ mm]

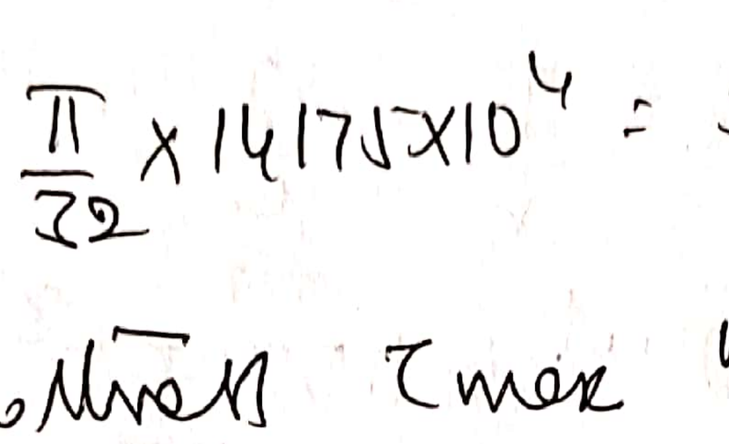
$= \frac{5567510 \times 120}{\frac{\pi}{32} \times 14171 \times 10^4} = 46$ N/mm^2 ✓

Angle of twist per unit length $= \frac{T}{GJ} = \frac{5566510}{85000 \times \frac{\pi}{32} 14171 \times 10^4}$.

$\therefore \frac{\theta}{L} \Phi = 4.7 \times 10^{-4}$ rad/mm

$= 0.0047$ rad/m $= 0.27°$/m.





$\frac{36}{90} = \frac{AB}{120}$.

$AB = \frac{36}{90} \times 120 = 48$ N/mm²

Comparison of wt of solid and hollow shafts.

3 Mb) Compare the weight per metre length of a solid circular shaft that of a hollow circular shaft (Inside dia being 70% of outside dia) if they are made of same material, same length & have same torsional resistance.

Sol: For solid circular shaft, Torsional resistance = $\frac{J}{r}\tau$

r is the radius of shaft, $J = \frac{\pi d^4}{32}$

Torsional resistance = $\frac{\pi r^4 \cdot \tau}{2r} = 1.571 \tau r^3$

For hollow circular shaft, outside dia, d_o & Internal dia

Polar moment of Inertia $J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(d_o^4 - (0.7d_o)^4)$

$J = \frac{0.074}{+.194} d_o^4$, $\frac{J}{r} = \frac{J}{\frac{d_o}{2}} = \frac{0.074 \times 2 d_o^4}{d_o} = 0.147 d_o^3 \tau$

& Torsional resistance are equal ie $1.571 \tau r^3 = 0.147 d_o^3 \tau$

$\frac{r^3}{d_o^3} = \frac{0.147}{1.571} \Rightarrow \frac{r}{d_o} = \sqrt[3]{\frac{0.147}{1.571}} = 0.455$

$\therefore$ outer radius of hollow shaft $r_o = 0.455 \times 2r$ $\quad \frac{2r}{d_o} = 0.91$

$\therefore r_o = 0.7 r_o = (0.7 \times 0.91) r_o$

for same material, same length & have same torsional resistance

$\text{Wt/m} \propto \text{Area}$ ie $\frac{\text{Wt of solid shaft}}{\text{Wt of hollow shaft}} = \frac{\text{Area of solid}}{\text{Area of hollow}}$

$= \frac{\pi r^2}{\pi[(0.91r)^2 - (0.637r)^2]} = \frac{r^2}{0.422 r^2} = 2.36$

(8) Find the polar modulus, torsional strength & torsional moment resistance of hollow circular shaft of 120 mm & internal dia of 70 mm thick. $q = 90$ GPa & permissible shear stress = 60 N/mm²

Sol: Internal dia, $d_i = 120$ mm. External dia $d_o = 120 + 10 + 10 = 140$ mm

Polar MI of Section, $J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(140^4 - 120^4) = 17.35 \times 10^6$ mm⁴

Section modulus $Z_p = J/r = 17.35 \times 10^6 / 70 = 0.25 \times 10^6$ mm³.

Torsional strength $= qJ = 90000 \times 17.35 \times 10^6 = 1561.5 \times 10^9$ N-mm

Torsional moment of resistance $= Z_p \times \tau = 0.25 \times 10^6 \times 60 = 15 \times$

Problem: A hollow circular steel shaft of inner radius 30mm is subjected to a torque of 10KN-m. Determine (a) max & min shear stresses developed in shaft (b) angular twist over 1m length of shaft. $G = 82 KN/mm^2$

outer radius $R_2 = 50mm$.

Inner radius $R_1 = 30mm$.

Polar moment of inertia

$$J = \frac{\pi}{2}(R_2^4 - R_1^4)$$

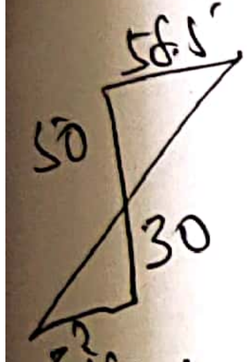
$$= \frac{\pi}{2}(50^4 - 30^4) = 854.5 \times 10^4 mm^4.$$

Torque $T = 10 KN-m = 10 \times 10^6 N-mm$.

Max. shear stress $\tau_{max} = \frac{T}{J} \times R_2 = \frac{10 \times 10^6}{854.5 \times 10^4} \times 50 = 58.5 N/mm^2$

Min. shear stress $\tau_{min} = \frac{10 \times 10^6}{854.5 \times 10^4} \times 30 = 35.1 N/mm^2$

$\Rightarrow$ (or) $\tau_{min} = \frac{58.5}{50} \times 30 = 35.1 N/mm^2$

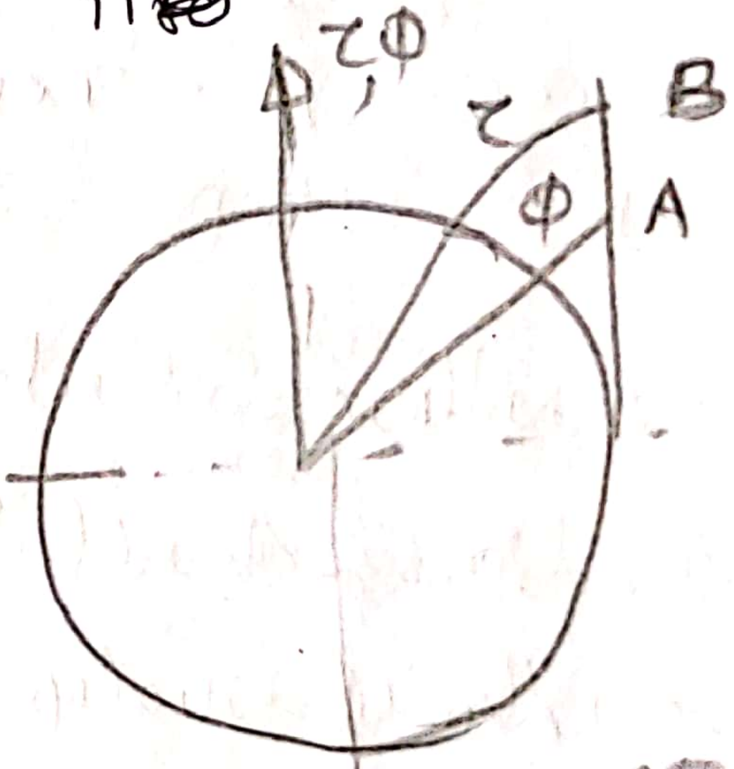


(b) Angular Twist for $L = 1m = 1000mm$.

$$\theta = \frac{TL}{GJ} \quad \left[\because \frac{T}{J} = \frac{G\theta}{L}\right] \Rightarrow \theta = \frac{10 \times 10^6 \times 1000}{82 \times 10^3 \times 854.5 \times 10^4}$$

$\theta = 0.01427 rad \Rightarrow 0.01427 \times \frac{180}{\pi} = 0.818^\circ$.

$\left[\because \pi rad = 180^\circ \Rightarrow 1 rad = \frac{180^\circ}{\pi}\right]$



Shear Stress - Strain curves

Modulus of Rupture: Torsion formula $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$ has been derived on the assumption that shear stress is proportional to shear strain, but when specimen is tested in Torsion it fails by fracture and stress developed in the material lies beyond the proportional limit, ie in plastic Region.

If shear stress & shear strain is drawn along radius of shaft shear strains at any point remain proportional to radius but τ at any point is no longer proportional to its radius, rather

the curve becomes non linear. Let T_{max} = Max. torque @ which shaft fails when test piece is continuously twisted.

Max. shear stress $\tau_{max} = \frac{T_{max}}{\pi R^4} \times 2R = \frac{2T_{max}}{\pi R^3}$ $\left[\because \frac{T}{J} = \frac{\tau}{R}\right.$

where $J = \frac{\pi R^4}{2}$ = Polar moment of inertia $\left[\because \tau = \frac{T}{\frac{\pi R^4}{2}}\right.$

Pg 74 Pr 4pm

A steel sample of a gauge length of 250 mm is tested under torsion and it was observed that fracture occurred at the torque of 920 N-m. Determine modulus of rupture if the diameter of the specimen is 25 mm.

Sol: Max torque, $T_{max} = 920$ N-m $= 9,20,000$ N-mm

Shaft Radius, $R = 12.5$ mm.

Modulus of rupture, $\tau_{max} = \frac{2T_{max}}{\pi R^3} = \frac{2 \times 9,20,000}{\pi \times 12.5^3}$

$= 300$ N/mm^2.

Horse Power Transmitted by Shaft:

Let N = No. of Revolution per min of shaft

T = Average Torque in KN-m

W.D/min = Force x Distance $= T \times 2\pi N$.

Power = W.D/sec $= \frac{2\pi NT}{60}$ KN-m.

(or) $P = T \times \omega = T \times \frac{2\pi N}{60}$ → where ω = angular velocity

But 1 H.P = 746 W, ∴ H.P transmitted by shaft =

Pg 75 A hollow circular shaft is transmitting power @ 300 rpm. The outer dia of shaft is 80 mm & inner dia is 70 mm. The max shear stress developed in shaft section is 65 N/mm^2. Determine power transmitted by shaft.

Sol: outer Radius $R_2 = 40$ mm, inner Radius, $R_1 = 35$ mm, rpm

$J = \frac{\pi(R_2^4 - R_1^4)}{2R_2} = \frac{65 \times \pi(40^4 - 35^4)}{2 \times 40} = 5,53742$ N-

$= 5.5374$ kN-

$= 5.537.4$ N-

HP Transmitted $= \frac{2\pi NT}{60 \times 746} = 233.2$ H.P.

Power and Torque: A shaft is said to torque when power transmitted through various means to serve some useful purpose. The many attachments used for this purpose affect the shaft by making cuts and holes in the shaft. This makes the shaft weak.

If a shaft is rotating at N revolutions per minute, the avg torque being T, N-m, W.D = Avg torque x angle of turned in 1 rev

Power, P = W.D/s = $T \times \frac{2\pi N}{60}$

$\therefore$ Avg torque = $\frac{60 P}{2\pi N}$ N-m

If power is in H.P, then P = P x 750 (or) P x 746

$\therefore$ $T_{avg} = \frac{60 \times P \times 746}{2\pi N}$.

The max. torque may be above the average value by 20 to 40%. The shaft has to be designed for max. torque. The design torque is always more than avg torque for taking in to account the weakness by cut a holes drilled for attaching gears, pulleys etc

Problem: A hollow circular steel shaft is required to transmit 300 HP at 200 rpm. The max. torque developed is 1.5 times mean torque. Determine the external diameter of the shaft if it is double the internal diameter. The max. shear stress not to exceed 80 MPa. Given $G = 82 kN/mm^2$.

Sol: Given H.P = 300, P = 300 x 746 N-m/s.

N = 200 rpm, $\omega = \frac{2\pi N}{60} = 20.94$ rad/s.

Mean torque, $T_{mean} = \frac{300 \times 746}{20.94} = 10,685.6$ N-m

Dia. Hor. torque, T_{HOR} = E.F. T_{HOR} = 1.5 × 10,685.46

$\therefore T_{HOR}$ = 16,024.5 N-M = 16,024.5 × 10^3 N-MM

Let External dia = d_0 $\therefore d_i = 0.5d_0$

J = polar moment of inertia = $\frac{\pi}{32}[d_0^4 - d_i^4]$

$\therefore J = \frac{\pi}{32}[d_0^4 - (0.5d_0)^4] = \frac{\pi}{32} d_0^4 (1 - 0.624)$

Hor shear stress, τ = 80 MPa @ radius $d_0/2 = r$

$\therefore \frac{T}{J} = \frac{\tau}{r} \Rightarrow \frac{T}{J} = \frac{\tau}{0.5d_0}$

$\frac{16024.5 \times 10^3}{0.092 d_0^4} = \frac{80}{0.5 d_0} \Rightarrow \frac{d_0^4}{d_0} = \frac{16024.5 \times 10^3}{80 \times 0.092}$

$\Rightarrow d_0^3 = 1088892.60 \Rightarrow d$ = 98.22 mm

Q.C
Q4
Pr. A vessel having a single propeller shaft 200 mm in dia runs at 300 rpm & required to run propeller shaft & equal dia producing 40% more HP @ 500 rpm. If the working shear stress in recquired shafts is 15% more than in single shaft, determine diameter of these shafts.

Sol: Say T_1, Torque transmitted by a single shaft

T_2, Torque transmitted by each of two shafts

$H_2 = 1.4 H_1$ (given) $\therefore H_1 = \frac{2\pi \times 300 \times T_1}{60 \times 746} \Rightarrow T_1 = 23.75 H_1$

H_2 by one shaft = $\frac{1.4}{2} H_1 = 0.7 H_1$., Now, $T_2 = \frac{0.7 H_1 \times 60 \times 746}{2\pi \times 500}$

$\frac{T_1}{T_2} = \frac{23.75 H_1}{9.97 H_1} = 2.38$., Here $T_1 = \frac{\pi}{16} \tau d_1^3$, $T_2 = \frac{\pi}{16} \tau' d_2^3$

Where $\tau' = 1.15\tau$. $\therefore \frac{T_1}{T_2} = \frac{d_1^3}{d_2^3} \times \frac{\tau}{\tau'} \Rightarrow 2.38 = \frac{200^3}{d_2^3}$

$\therefore d_2^3 = \frac{200^3}{2.38 \times 1.15} = 2.92 \times 10^6 \Rightarrow d_2 = 142.97$ mm = 143

Dia of each recquired shaft = 143 mm.